MAT 362 SPRING 05 HOMEWORK 7

Due Thursday, April 6

1. Do Problem 7.12, p.154 in the text. Even though its done quite completely in the solution section, please go through it carefully and make sure you understand all the steps.

2. Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be the mapping given by

$$F(p) = \lambda p,$$

where λ is a non-zero constant. This is a "similarity" transformation of $\mathbb{R}^3$.

Let S be a (regular) surface in $\mathbb{R}^3$ and let $\widetilde{S} = F(S)$.

(a). Show that $\widetilde{S}$ is also a regular surface.

(b). Find formulas relating the Gauss and mean curvatures, $\widetilde{K}$ and $\widetilde{H}$, of $\widetilde{S}$ with the Gauss and mean curvatures, K and H, of S.

3. Describe the image of the Gauss map of the following surfaces, i.e. what is the region in S^2 that the surface maps to under the Gauss map.

(a). Paraboloid of revolution: $z = x^2 + y^2$.

(b). Hyperboloid of revolution: $x^2 + y^2 - z^2 = 1$.

(c). Catenoid: $x^2 + y^2 = \cosh^2 z$.

4. Do Problem 7.19, p.169 of the text.

5. (Harder) Suppose S is a minimal surface, i.e. $H = 0$ without umbilic points. Show that the Gauss map $N : S \to S^2(1)$ satisfies, for all $p \in S$,

$$\langle dN_p(w_1), dN_p(w_2)\rangle_{N(p)} = \lambda(p)\langle w_1, w_2\rangle_p.$$

Here w_1, w_2 are vectors in T_pS and λ is a function on S. Explain why this means the Gauss map is a conformal map of surfaces.