MAT 362 SPRING 05 HOMEWORK 5

Due Thursday, March 9

1. Suppose S is a regular surface in $\mathbb{R}^3$ given as the 0-level set of a smooth function, $S = \{f(x, y, z) = 0\}$. Assume 0 is a regular value of f.

Show that the vector

$$N = \frac{\nabla f}{||\nabla f||},$$

is a unit vector everywhere orthogonal to S. Hint: compute the dot product of any tangent vector to S with N.

Conclude that S is orientable.

2. Do Problem 5.4, p.100 of the text. You can use the solution in the back as a hint, but do this problem in detail.

3. Do Problem 5.8, p.106 of the text. Again, you can use the solution in the back as a hint, but do this problem in detail.

4. A torus of revolution has a coordinate chart given by

$$\sigma(u, v) = ((a + r\cos u)\cos v, (a + r\cos u)\sin v, r\sin u),$$

where $u, v \in (0, 2\pi)$. Here r and a are constants. Find the first fundamental form of this torus in this coordinate chart.

5. On the surface with coordinate patch given by

$$\sigma(u, v) = (u\cos v, u\sin v, \log\cos v + u),$$

where $v \in (-\frac{\pi}{2}, \frac{\pi}{2})$, compute the lengths of the curves $\sigma(u, v_0)$, where v_0 is any constant and u varies from u_1 to u_2. The answer should be independent of v_0.