MAT 203, Fall 2001
Final Examination

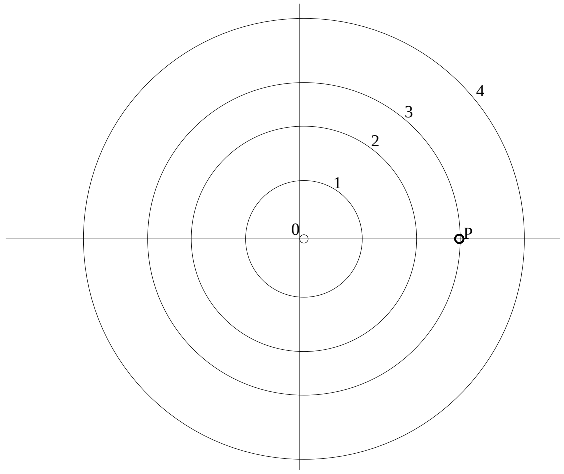
December 17, 2001

This examination consists of three parts, A, B and C. Answer all four questions in part A, four of the five in part B and one of the two in part C (You may choose to answer more questions - the best four in part B and the best one in part C will be counted).

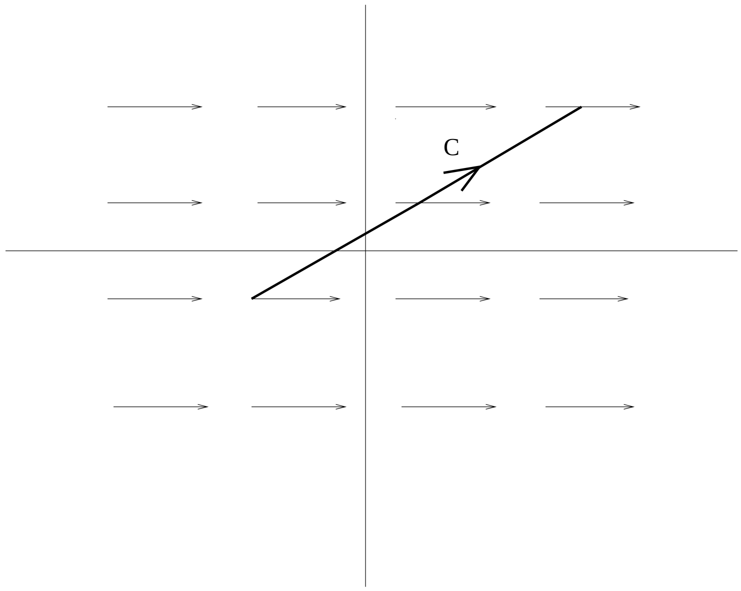
Part A

In each of the following, answer the question with 'Yes' or 'No' only. A correct answer gets 5 points and an incorrect one -3 points. No partial credit will be given.

1. The contour diagram of $f(x, y)$ is below. Is the directional derivative $f_{\hat{u}}(x, y)$ at the point P in the direction $\hat{u} = (1/\sqrt{2}, 1/\sqrt{2})$ positive?



2. Consider the vector field $\vec{F}(x, y)$ sketched below. Is the line integral $\int_C \vec{F}$ positive?



3. Are the vectors $\hat{\imath} + \hat{\jmath}$, $\hat{\jmath} + \hat{k}$ and $\hat{k} - \hat{\imath}$ coplanar (i.e., contained in a plane).

4. Is the vector field $xy\hat{\imath} + y^2\hat{\jmath}$ the gradient of a function?

Part B

Answer any four of the questions. Each question is worth 15 points.
Show all work You must justify your answers

1. Let $\vec{u} = 3\hat{i} + \hat{j}$ and $\vec{v} = \hat{i} + \hat{k}$ be two vectors.

   (a) Find a unit vector $\vec{w}$ perpendicular to $\vec{u}$ and $\vec{v}$.

   (b) Find the equation of the plane through $(1, 0, 3)$ perpendicular to $\vec{w}$

2. (a) Find all the critical points of the function $f(x, y) = x^3 - xy + 3y^2$

(b) Deduce whether these are minima, maxima or saddle-points.

3. Let R be the region in the plane above the curve $y = x^2$ and below the line $y = x$.

   (a) Express $\iint_R xydxdy$ as an iterated integral.

   (b) Interchange the order of integration in the above.

   (c) Evaluate the integral.

4. Let R be the unit disc in the plane.

(a) Express the integral $\iint_R (x^2 - y^2) dxdy$ as an integral in polar coordinates.

(b) Evaluate this integral (using the identity $cos(2\theta) = cos^2(\theta) - sin^2(\theta)$)

5. Find the flux integral of $\vec{F}(x, y, z) = xz\vec{i} + sin(x)\vec{k}$ on the vertical cylinder $\{(x, y, z) : x^2 + y^2 = 1, 0 \leq z \leq 2\}$

Part C

Solve one problem. Each problem is worth 20 points,
Show all work. You must justify your answers

1. Using Lagrange multipliers, find the points on the surface $xyz = 1$ that are closest to the origin. (**Hint:** Minimise the square of the distance from the origin)

2. Let $\vec{F}(x,y) = (e^x + 2xy)\hat{\imath} + (cos(y) + x^2)\hat{\jmath}$.

(a) Let C_1 be the line segment joining $(0,0)$ and $(1, e-1)$. Compute $\int_{C_1} \vec{F} \cdot \vec{dr}$

(b) Let C_2 be the segment of the curve $y = e^x - 1$ joining $(0,0)$ to $(1, e-1)$. Using Green's theorem, deduce the value of $\int_{C_2} \vec{F} \cdot \vec{dr}$