MAT 342: Homework 5

INSTRUCTIONS: Please read carefully the following instructions for this assignment.

1. You have time to submit this assignment until August 10, you need to do by uploading it in **Gradescope**. Unless you can do it by writing it down in a tablet, use an app to scan it (like CamScanner, not just pictures) and send it in .pdf format.
2. When you submit your assignment, indicate what pages correspond to what problem in your submission.
3. This an assignment, so you can discuss these problems, but in case you do that, please make sure that the solutions that you send me are in your own words. This is our chance to correct any mathematical mistake that you do when writing.

Problem 1. *Find the singularities of the following functions. If $a \in \mathbb{C}$ is an avoidable singularity of f, find the value of $f(a)$ so that f becomes holomorphic on a neighborhood of $a \in \mathbb{C}$. If $a \in \mathbb{C}$ is a pole, determine the order and the negative part of the Laurent series.*

$$\text{(a) } f(z) = z\cos(1/z) \qquad \text{(b) } f(z) = \frac{1}{(1-e^z)^2} \qquad \text{(c) } f(z) = \frac{z^2+1}{z^3(z-1)^2}$$

$$\text{(d) } f(z) = \frac{1+z}{1+\sin z} \qquad \text{(e) } f(z) = \frac{(z^2-1)^2}{z^2(z+1/2)(z+2)}.$$

In the following problems you will be computing integrals using residues. When doing so, remember to explain why the integrals you are considering are convergent.

Problem 2. *Compute*

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}.$$

Problem 3. *Compute*

$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx.$$

Problem 4. *Compute*

$$\int_0^{2\pi} \frac{\sin^2(t)}{5+4\cos(t)} dt.$$

(Hint: You will need to make some modifications to reduce the problem to computing the residues of a multiple of the function in Problem 1 (e))

Problem 5. *For $a \in (0,1)$, compute*

$$\int_0^{\infty} \frac{x^a}{1+x^2} dx.$$

Problem 6. *Compute*

$$\int_0^{\infty} \frac{\log x}{1+x^2} dx.$$