MAT 342: Homework 3

INSTRUCTIONS: Please read carefully the following instructions for this assignment.

1. You have time to submit this assignment until July 29, you need to do by uploading it in **Gradescope**. Unless you can do it by writing it down in a tablet, use an app to scan it (like CamScanner, not just pictures) and send it in .pdf format.
2. When you submit your assignment, indicate what pages correspond to what problem in your submission.
3. This an assignment, so you can discuss these problems, but in case you do that, please make sure that the solutions that you send me are in your own words. This is our chance to correct any mathematical mistake that you do when writing.
4. You will get a grade from 0 to 5 depending on how many problems you attempt.

Problem 1. *Study the convergence of the following power series and find the sum.*

$$(a) \sum_{n\geq 1} n^2 z^n \qquad (b) \sum_{n\geq 0} \frac{z^n}{(n+1)(n+2)} \qquad (c) \sum_{n\geq 0} \frac{z^{3n+1}}{3n+1} \qquad (d) \sum_{n\geq 0} \frac{z^{3n+2}}{3n+2}$$

Problem 2. *Let* $f(z) = (1+z)/(1-z)$.

(a) *Prove that there exists a determination of* $\log(f(z))$, $z \in \mathbb{D}$.

(b) *Find the power series expansion around* 0 *of this determination. Find the radius of convergence.*

Problem 3. *Suppose the radius of convergence of the power series* $\sum a_n z^n$ *is* 2 *and that* k *is an integer. Compute the radius of convergence of the series*

$$\sum a_n^k z^n.$$

Problem 4. *Consider the domain* $\Omega = \mathbb{C} \setminus (-\infty, 1]$ *and the determination of* $\sqrt{z^2-1}$ *in* Ω *that is positive in* $(1, \infty)$.

(a) *Show that* $z + \sqrt{z^2-1}$ *omits the negative real axis, thus the principal determination of* $Log\left(z + \sqrt{z^2-1}\right)$ *is well-defined in* Ω.

(b) *Prove that* $Log\left(z + \sqrt{z^2-1}\right)$ *is a primitive of* $\left(\sqrt{z^2-1}\right)^{-1}$ *in* Ω.

Problem 5. (a) *Find all the zeros of* $\cos(z)$ *and* $\sin(z)$.

(b) *Show that* $|\sin z|^2 = |\sin(x+iy)|^2 = \sin^2(x) + \sinh^2(y)$. *Over what lines is* $\sin z$ *a bounded function?*