

# Elias M. Stein (1931–2018)

*Lillian B. Pierce*

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Elias M. Stein (1931–2018) had a profound influence on the field of analysis. He developed tools that are now indispensable, expanded and clarified major theories, and introduced new classes of questions that continue to stimulate research today. In addition, his singular skills as a mentor and expositor left a legacy of dozens of PhD students, hundreds of mathematical descendants, and thousands of loyal readers.

Elias Menachem Stein was born in Antwerp, Belgium, on January 13, 1931. His parents, Elkan Stein and Chana Goldman, both Polish citizens, fled Belgium with their children after the German invasion in 1940. Nine-year-old Elias made part of the journey with diamonds in the soles of his shoes, where Elkan, a diamond merchant, had concealed them.

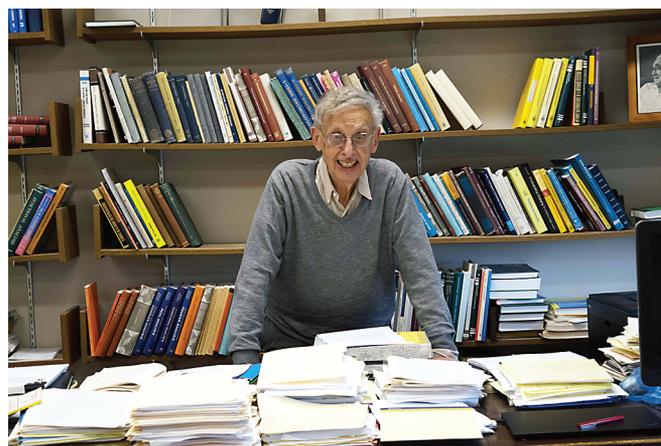
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**Figure 1.** Elias M. Stein in his office in Fine Hall, Princeton University, 2017.

The family arrived in the United States in the spring of 1941, and settled in New York. Stein attended Stuyvesant High School, where he served as captain of the math team, and graduated in 1949. He then attended the University of Chicago, where he earned a Bachelor's degree in 1951, a

Master's in 1953, and a PhD in 1955, with a thesis advised by Antoni Zygmund, titled *Linear operators on  $LP$  spaces*. Stein was then an Instructor for two years at MIT, before taking up in 1958 an assistant professorship at the University of Chicago, where he was swiftly promoted.

After a year at the IAS in 1962–1963, Stein joined the Department of Mathematics at Princeton University as a full professor in 1963. From 1975–2012 he was the Albert Dod Professor of Mathematics at Princeton University. In 2012, he transferred to emeritus status, although he continued to teach popular courses at Princeton. Stein died on December 23, 2018 at 87 years old, from complications related to mantle cell lymphoma.

Elias Stein is survived by Elly, his wife of 59 years. Their son Jeremy is the Moise Y. Safra Professor of Economics and Chair of the Economics Department at Harvard University. Their daughter Karen is an architecture critic and a former member of the jury of the Pritzker Architecture Prize.

Over the course of his long career, Elias Stein developed many powerful tools used widely in harmonic analysis today—as just a few examples, Stein's interpolation theorem, the Stein maximal principle, the Cotlar-Stein lemma. In addition, Stein had remarkable intuitions for the interactions between wide-ranging fields: Fourier analysis, complex function theory, partial differential equations, real analysis, geometry, number theory. When Stein was awarded the Wolf Prize in 1999, the citation emphasized the role of such connections in his work [3]: “Elias M. Stein has made fundamental contributions in mathematical analysis understood in a very broad sense. He developed (jointly with G. Weiss and C. Fefferman) the theory of Hardy spaces in several real variables; in particular, this emphasized the role of duality between the Hardy spaces and the BMO spaces introduced earlier by F. John and L. Nirenberg. In the representation theory of Lie groups, Stein discovered, with R. Kunze, the so-called Kunze-Stein phenomenon, classical by now, regarding harmonic analysis and certain nonunitary representations of semisimple Lie groups. Stein also made profound contributions to the  $\bar{\partial}$ -problem of several complex variables. He is one of the creators of multidimensional Euclidean Fourier analysis, having shaped classical analysis by recognizing the role of singular integrals, Radon transforms, and maximal operators obtained by integration on lower-dimensional manifolds in Euclidean spaces.”

Stein asked questions that opened new areas. For example, his questions about the interaction of the Fourier transform and curvature continue to stimulate research in the broad and fascinating area surrounding restriction problems. He initiated the study of polynomial Carleson operators. And his late-career investigation of discrete

operators, at the intersection of harmonic analysis and number theory, popularized a young area of analysis.

A full perspective on Stein's immense research program can be found in the retrospective appreciation [2].

In 2011, Princeton University honored Stein's 80th birthday with the event “Analysis and Applications—A Conference in Honor of Elias M. Stein.” In addition to a full slate of research talks, this conference featured a discussion titled *Mentoring, Collaborating, and Writing Books*. In each of these three areas, Stein excelled.

Stein was a prolific advisor, mentoring at least 52 PhD students, including the Fields medalists Charles Fefferman and Terence Tao. The Mathematics Genealogy Project lists approximately 550 descendants. At the time of Stein's transfer to emeritus status in 2012, his colleagues at Princeton University stated: “Eli's combined influence as a researcher, collaborator, teacher and expositor is unmatched. His lectures are characterized by perfect clarity, concentration on essentials and impeccable taste. In his interaction with students and coworkers he has managed to convey the strong sense of optimism that is essential for mathematical discovery. He has been a major influence on many lives.”

Stein was also well-loved as an instructor of undergraduate courses at Princeton. This was recognized by Princeton University's Award for Distinguished Teaching in 2001, and he continued to teach undergraduate courses with cult status until he was 86.

Stein's bibliography comprises 234 publications, including 15 books and monographs. He worked with over 60 collaborators, many of whom have contributed recollections to this memorial.

The “Stein school” expands far beyond collaborators and direct descendants, to mathematicians all over the world who treasure his books upon their shelves. Stein's books are prized for their clear and far-ranging treatments of foundational topics in harmonic analysis. These include *Singular Integrals and Differentiability Properties of Functions* (1970), which won the Leroy P. Steele Prize in 1984, *Fourier Analysis on Euclidean Spaces* (1971), coauthored with Guido Weiss, and the iconic *Harmonic Analysis: Real Variable Methods, Orthogonality, and Oscillatory Integrals*. These vivid accounts of the field of harmonic analysis are animated by the central role that Stein played in the field's development. As he wrote in the preface to *Harmonic Analysis*, “...I cannot deny that this book is in part autobiographical: as the narrator of the story, I have chosen to recount those matters I know best by virtue of having first-hand knowledge of their unravelling.” Finally, in his 70s, Stein collaborated with Rami Shakarchi on writing *The Princeton Lectures in Analysis*, a celebrated four-book series on Fourier analysis, complex analysis, real analysis, and functional analysis.



**Figure 2.** The Stein family at the White House, for the National Medal of Science ceremony in 2002. From left to right, Karen, Elly, Eli, Jeremy, and Anne, with grandchildren Carolyn, Jason, and Alison in the front.

Stein was chosen as a Sloan Fellow in 1961 and was invited three times to speak at the ICM (1962, invited speaker; 1970, 1986, plenary speaker). He was a Guggenheim Fellow in both 1976–1977 and 1984–1985 and in 1989 he was awarded a von Humboldt Award. Stein was a Member at the IAS in 1962–1963, 1976–1977, and 1984–1985.

In recognition of his profound influence on the field of analysis, Stein was awarded the Schock Prize from the Royal Swedish Academy of Sciences (1993), the Wolf Prize in Mathematics from the Wolf Foundation in Israel (1999), the Leroy P. Steele Lifetime Achievement Award from the AMS (2002), the Stefan Bergman Prize of the AMS (2005), and a National Medal of Science (2002).

Stein was a member of the National Academy of Sciences (1974), a member of the American Academy of Arts and Sciences (1982), and an inaugural Fellow of the AMS (2013). He was awarded honorary degrees from Peking University (1988) and the University of Chicago (1992).

At Princeton, Stein served twice as chair of the Mathematics Department, in 1968–1971 and 1985–1987. He was also the editor of the *Annals of Mathematics Studies* book series at Princeton University Press for nearly 40 years. David Gabai, the Hughes-Rogers Professor of Mathematics and Chair of the Mathematics Department at the time of Stein's death, stated: "By so many different measures Elias Stein was truly an incredible and extraordinary mathematician. As a research mathematician, he did incredible and influential work. He was an extraordinary mentor; many of his PhD students have become extraordinary leaders in mathematics. As an undergraduate teacher, he was extremely popular with students. And as a citizen, he was an essential figure in all aspects of the math department for many years."

Stein will be remembered for his delight in good problems, generosity, and perpetual optimism. As he wrote in his response to the Leroy P. Steele Lifetime Achievement Award, "We can be confident that we are far from the end of this enterprise and that many exciting and wonderful theorems still await our discovery."

### Harold Widom

Eli was a class ahead of me at Stuyvesant High School, captain of the math team, and far ahead of the rest of us mathematically. It was amazing how much mathematics he knew at the time. I thought of him as my mentor. He recommended Titchmarsh's *Theory of Functions* which I was lucky enough to find at the local Barnes and Noble (for five dollars). I still have my tattered copy. I'm quite sure it was because of this that I eventually went into analysis. Eli graduated before I did and although we never worked jointly our paths crossed several times. First, when we were both graduate students at the University of Chicago. He was a student of Antoni Zygmund (in analysis, of course) and although I thought I might follow in his footsteps I eventually wrote my thesis in functional analysis with Irving Kaplansky. We were together again at Chicago in 1962–1963 when he was on the faculty and I was a visitor the year they had a program in analysis. (I'm sure he was instrumental in getting me invited.) We met several times at conferences and again in 1987 when I was at the Princeton Institute and Eli was on the Princeton faculty. By then our mathematical interests had diverged but he and Elly were generous with their time and hospitality. He was a fine person and great mathematician, and will be missed.

### Stephen Wainger

When I was a graduate student, I learned a great deal from Eli both from courses he taught and from informal conversations. All of this had a tremendous influence on my career. Before I met Eli, I just memorized proofs of theorems. Eli emphasized to me the importance of trying to understand the ideas behind proofs. Also in informal conversations he taught me the method of stationary phase, and in his course on analytic number theory, I learned the circle method. These techniques were basic in my research work.

Eli also suggested problems for us to work on together. At a meeting in Chicago, he suggested we work on the problem of finding bounds for the Hilbert transform along

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curves. Work on operators along curves became one of my central areas of research, and of course the method of stationary phase is a basic tool in this study. Another subject that Eli suggested we work on was that of discrete problems. This also became a central theme of my work, and the circle method which I had learned in his class was a basic tool here.

In summary, I owe my career to Eli's influence.

## William Beckner

I met Eli Stein in May 1969—I drove out to Princeton from our farm in Missouri to look over the campus and meet faculty. This was the last term that the department would be in Fine Hall—the new Fine Hall would open in September so there was still an opportunity to see the magnificent common room and sit in the windowed nooks in Fine Library. I can still recall walking through the darkened lower floor to find Eli's office—as he threw open the door, the warm light in his eyes, and his enthusiasm for mathematics would in part inspire the course of my future study.

That year would be memorable as new Fine Hall opened, Eli's classic book on *Singular Integrals* would go to press, his term as department chair would initiate a new sense of community within Princeton mathematics, and I would take the best math class of my life, an introduction to Fourier analysis and topics in analysis as only Eli Stein could present that subject. In this class, Eli told many stories about his years at Chicago which now I wish so much that they had been recorded. A familiar theme in his lectures was that the key step in an argument is to know when to make an estimate. Eli lectured in a casual manner, but he prepared carefully—always looking for the “right way” to explain essential points.

I found the department to be particularly warm toward grad students, an atmosphere perhaps inspired by the earlier return of Don Spencer to Princeton—and Eli in particular had a very generous spirit as I found myself often invited to have dinner at his home on Dodds Lane with Elly, and math parties too. Before starting grad school, I had met Steve Wainger in Madison and Charlie Fefferman at a conference in Montreal—both pushed strongly that Eli Stein was the best adviser. I did finish my dissertation in 1975, but I had no expectation of attending the formal ceremony in June on the lawn facing Nassau Hall. Eli had four students graduating that spring so he asked if we could all participate so that he could march with us. Somehow I was able to rent a gown, but Eli lacking a Chicago gown had to borrow Fred Almgren's gown

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from Brown. A special moment occurred for me with the arrival in the math office of a telegram from J-P. Kahane with news about the Salem Prize. In his typical style Eli immediately said “we must celebrate” so together with Joe Kohn and Si Kochen we drove to a favorite Chinese restaurant in Princeton Junction for an extended mid-day lunch.

On the personal side there was an immediate connection which was always present. Eli was broadly appreciative of design and setting—from the blackboard to the world at large—Siena was one of his favorite spots in Italy, he could with ease discuss Renaissance art and architecture, and at one point he suggested that I read Sartre's essay on Tintoretto. On the math side Eli bridged so much of analysis in quick understanding that he often seemed to his students and collaborators to be a monumental force of nature. Eli's mathematics has been a constant source of inspiration for me where I continually recognize mechanisms in his work that percolate through contemporary analysis, especially his characterization of functional smoothness using potentials, the identification of the Heisenberg group as the intrinsic manifold past Euclidean space that exemplifies the interconnection of analysis and geometry on a Lie group, the role of Kunze-Stein phenomena as a signpost for understanding convolution on a Lie group, and the surprising breadth of application for Stein-Weiss integrals. We enjoyed many wonderful travel adventures together, starting with a conference in the Villa Monastero in Varenna on Lago di Como and most recently the Tony Carbery birthday celebration in Edinburgh—always involving excellent dinners with good conversation and good mathematics.

Eli Stein was a vivid presence in my life and I miss him greatly.



**Figure 3.** Peter Greiner, Eli Stein, Gerald Folland, and Linda Rothschild in Cortona, 1976.

## Linda Rothschild

When I received an email message telling me that Eli had died, I was incredulous; I had thought he was immortal. I met him for the first time at a conference in 1972, just two years after I had finished my PhD at MIT and was still wandering through different research directions. I had heard much about him from young mathematicians recently trained in Princeton, who spoke of the amazing Professor Stein in almost reverential terms. To them, Eli was not only a great mathematician, but also the most remarkable teacher and expositor that ever lived. When I finally met Eli he did not seem like the godlike figure I had expected, but rather a youthful, modest, and friendly mathematician with a keen interest in everyone and their ideas. I was soon to learn that the secret of his success was to use his powerful intellect and depth of mathematical insight not to toil alone and vanquish others, but to educate, collaborate, and bring out the best from the many students, postdocs, and younger mathematicians with whom he worked.

In the course of writing a long paper with Eli I learned not only how to think like a mathematician, but also how to work and behave like one. When I was ready to give up on some aspect of a problem, he would gently urge me to try again. When I tended to be careless and sloppy in my thinking, he told me to work through every detail and understand the basis of any theorem I tried to use. After our paper was finished, Eli said that this problem had been ripe for solving and that one must carefully choose what problems to tackle. (Some people divide great mathematicians into problem solvers and theory builders and classify Eli in the latter group. Presumably the former choose problems that are hard and important but not necessarily ripe.) Eli also taught me to take a positive view. He never ranked mathematicians; he would just offer praise for people whose work he admired. Eli was also very conscious that mathematicians were human. If I said something he disapproved of, he had a way of indicating this without making me feel upset or humiliated. In later years he treated me with the courtesy and friendship accorded to the respected mathematician that he had helped me become.

Eli's commitment and loyalty to his students, coauthors, and mathematics itself were extraordinary. If I could have one last conversation with him, I would thank him for everything and express my regret that he was not immortal.

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## Steven G. Krantz

Eli Stein was my teacher, my mentor, and my friend. He taught me a lot of good mathematics, but he also taught me how to think about mathematics and how to function as a mathematician. Part of the reason that the Princeton mathematics graduate program is so successful is that the faculty are mathematical paradigms for the students. Stein filled that role beautifully.

When I first met Eli Stein he took me to lunch and then we sat down to talk about what I would work on for my thesis. He asked me what I was interested in and I said, "The multiplier problem for the ball." He immediately said that was too hard, and he gave me three other problems. Those other three problems turned out to be just as hard. One of them (the Hilbert transform along curves) was ultimately solved by Nagel, Riviere, and Wainger. Another (Cauchy integrals on Lipschitz curves) was solved by Calderón. The third one had to do with group representations and I never really understood it.

At some point I went to Stein dejectedly and told him that I couldn't make any headway on any of these three problems, and I didn't know what to do. He very generously reached into his filing cabinet and gave me a problem that he was planning to work on himself. It turned out to be just the right problem for me, I understood it immediately, and I solved it rather quickly.

Even though I was only a student and Stein was a legendary mathematician, when we worked together it was as though we were equals. We stood side-by-side at the blackboard and slugged things out together. It was wonderful and formative and made me the mathematician that I am today. I still remember when, at the end of one of these sessions, he told me to go home and try to prove a certain convexity theorem. I thought about it and realized that the proposed convexity theorem was false. But it got me thinking in the right direction and I ended up getting the result I needed and solving the problem. To this day I do not know whether Stein knew in advance that this was the way it was going to work out.

In 1973 Bob Fefferman and I asked Stein to teach a course on  $H^p$  spaces. He readily assented, and the course he created was amazing. Of course it was a huge effort for him, but he did the job with elegance and grace. My notes from the course are still valuable to me.

Elias M. Stein was a great inspiration for myself and for many others. He will be missed in ways too myriad to describe.

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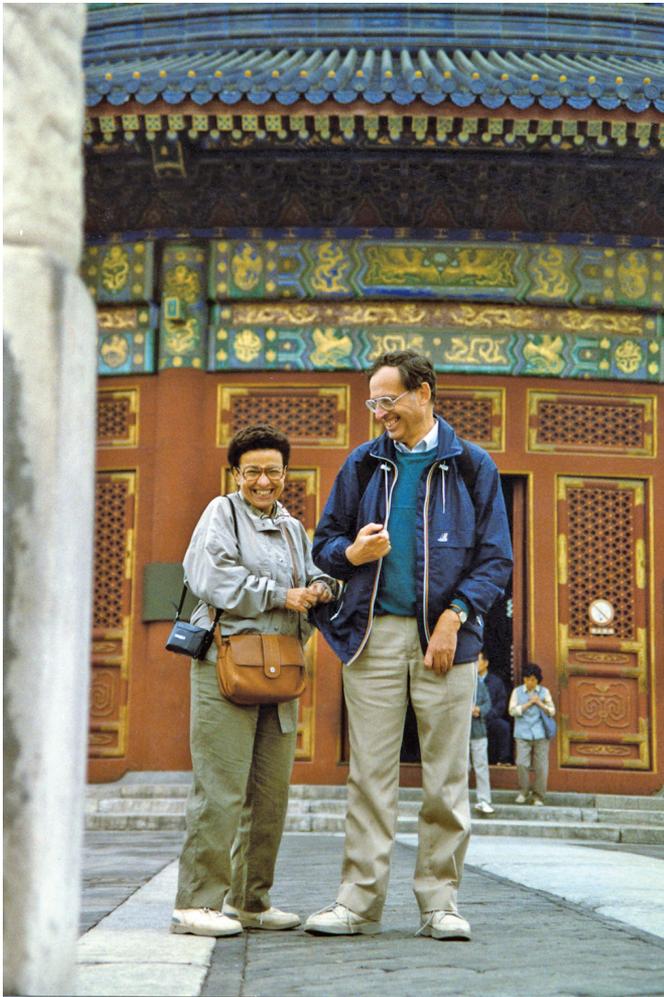


Figure 4. Elly and Eli Stein in Beijing, China, 1988.

## Alexander Nagel

In the fall of 1970 I was a new Instructor at the University of Wisconsin-Madison, and one afternoon while browsing in the University Book Store I happened upon a book, *Singular Integrals and Differentiability Properties of Functions*, by Elias M. Stein. At that time I knew essentially nothing about singular integrals, but since the price was only eleven dollars I bought it, and it changed my life. For me the book was an elegant exposition of the living, evolving field of harmonic analysis, with beautiful theory developed in the service of concrete problems. The writing style was friendly, and seemed to encourage even outsiders to engage with open questions. For example on pages 28–29 there is the following invitation: “The analogous study of kernels whose singularities are situated on

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varieties more general than isolated points is an important problem which it seems must be left to a future theory.” Who could resist? I couldn’t, and was lucky to be able to start working with Steve Wainger, Stein’s first student, who was my colleague at Madison.

I first met Eli Stein in the fall of 1975 when he invited me to spend a sabbatical year at Princeton. He had recently published two fundamental papers, which dealt with  $L^p$ , Lipschitz, and Sobolev space regularity of certain nonelliptic partial differential operators. The first paper with Jerry Folland dealt with the boundary Laplacian  $\square_b$  on strictly pseudoconvex domains, and the second paper with Linda Rothschild dealt with more general hypoelliptic operators  $\mathcal{L} = X_0 + \sum_{j=1}^p X_j^2$  studied by Hörmander. By locally approximating the given operator by an invariant differential operator on an appropriate nilpotent Lie group, one can use harmonic analysis on the group to construct a parametrix which is modeled by group convolution. This is a beautiful extension of the technique of freezing coefficients used in the study of elliptic operators. When I arrived in Princeton, Eli suggested, along these lines, that we try to develop a class of pseudodifferential operators which was broad enough to include some of these parametrices, and yet narrow enough so that operators of degree zero would still be bounded on interesting function spaces. Eli’s clear enjoyment of talking and doing mathematics soon overcame my initial nervousness, and during that first year I came to appreciate his instinct for knowing profitable directions for research and his talent for finding key problems to work on. He was able to see the mathematical forest, not just the trees.

Eli enjoyed working with people, and in 1984 he organized a summer school in analysis held at Peking University. I was invited, along with Rafi Coifman, Bob Fefferman, Carlos Kenig, Steve Krantz, Steve Wainger, and Eli himself, to deliver a series of lectures on recent developments in harmonic analysis. China had opened up after the Cultural Revolution, and I think Eli saw this as an opportunity to promote younger American mathematicians as well as to establish contact with and encourage a younger generation of Chinese mathematicians who might rebuild the profession after a decade of repression. In any case it was a great and successful adventure. We spent three weeks lecturing in Beijing, where we saw streets filled only with bicycles and horse-drawn carts. We then toured the country, spending a night in a military hospital in Xian, boating on the Li river, and ending at the luxurious White Swan Hotel in Guangzhou.

Eli enjoyed working with several people at the same time, and one of my most enjoyable collaborations involved Fulvio Ricci and Steve Wainger. Eli introduced me to Fulvio at an Oberwolfach meeting in 1989, and in

the mid-1990s we began to think about problems involving multiparameter harmonic analysis. Eli had for a long time been interested in situations and problems in which there are more than one natural dilation structure. Jointly with Peter Greiner he constructed a parametrix for the  $\bar{\partial}$ -Neumann problem for strictly pseudoconvex domains, which is the composition of standard isotropic operators and nonisotropic operators coming from the Heisenberg group. He pursued these ideas in work with D. H. Phong and later with Fulvio Ricci and Detlef Müller. The joint work with Fulvio, Eli, and Steve led to a study of flag kernels on nilpotent groups and most recently to the introduction of a class of singular integral operators controlled by multiple norms. The pleasure of the collaboration was enhanced by the setting, which was a long-running series of annual or semiannual meetings that rotated between Van Vleck Hall in Madison, Eli's office and the new Lewis Science Library designed by Frank Gehry at Princeton, and the elegant Palazzo della Carovana at the Scuola Normale Superiore di Pisa.

I cannot end this brief reminiscence without mentioning some nonmathematical memories. After my first visit to Princeton in 1975, my wife and I became good friends with Eli and his wife Elly. They helped us get settled with our very young children on our first visit to Paris in 1982 and they visited us in Madison and at our family vacation home in Vermont. During a summer visit to MSRI we toured the Napa Valley with them. When my older daughter was deciding where to go to college, she visited the Steins, and Eli talked about the beauty of the Princeton campus and its distinguished faculty. But when asked about his own college experience, he began to sing the praises of the University of Chicago where as an undergraduate he enjoyed literature, social science, history, and politics in addition to mathematics. My daughter ended up at Chicago, and I think Eli was pleased.

I knew Eli Stein for more than forty years as mentor, collaborator, and friend. To think about his passing brings pain and sadness but also brings good memories. I am sure it does for the many people who knew him.

### *Anthony W. Knapp*

Eli Stein arrived in Princeton for the Fall 1963 semester as I was beginning my second year in graduate school. I attended his course, which began by introducing the Fourier transform in Euclidean space and taking it from there. I enjoyed it. Sometimes I would call out a remark in class, and he would whirl around and say "What!" I would ask

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**Figure 5.** Eli Stein, Alex Nagel, and Elly Stein in Vermont, 2016.

my question or make my point, and he would continue. That kind of exchange established our relationship.

There was never a question of my becoming a student of his, since I was already on a track to become a student of Bochner or Feller. But in four semesters I took five courses from him. One of the courses was a general introduction to group representations, the kind of course that has been made into books by various authors over the years. I graduated and came back to Princeton as a member of the Institute for Advanced Study for the 1968–1969 year. Having learned about Lie groups in the meantime, I resolved to absorb more Stein mathematics. This time I followed him to his office after class quite often and asked him questions. I had realized that the orthogonal projection from the space of square integrable functions on the unit sphere in two complex variables to the subspace of functions that extend to be holomorphic in the ball had a group-theoretic interpretation with a certain Lie group and that the associated involution appeared to be similar to the operators of Calderón and Zygmund in Eli's courses. The new feature was that the underlying group was a nilpotent Lie group, not Euclidean space. Meanwhile in his research earlier

with Ray Kunze, Eli had investigated some singular integrals that arise in representation theory, and he tried to interest me in them. We soon realized that the two problems were the same, and there began a collaboration on “intertwining operators for semisimple groups” that continued for a dozen years.

Early on, he made it clear that we were doing this work as colleagues, not as a teacher and student. He laid out certain expectations, such as that I not bring a third person into the project without consulting him. I had no illusions that it would be an equal partnership. But it was a good collaboration: Eli had perspective and a knowledge of what other people had done, and I had patience and available time. He was very quick. The ten years’ age difference between us was an advantage.

We worked intensively on what turned out to be a problem for “principal series representations” for the first year. Several times he came up with an important idea in an oversimplified context, and he left it to me to dress up the result in terms of our setting with Lie groups. Together we gained control of so many examples that results were transparent even when general proofs were not.

The work continued through 1971, but Eli was losing interest and said as much, the work having become too algebraic for his taste. For Spring 1972 he was to be on leave in France at the IHES twenty miles from Paris, and he arranged for the university at nearby Orsay to invite me as an exchange professor for one month.

During that month Eli undertook to educate me about a style of living. I had never been to Europe before. Although he and his wife Elly were staying in the housing for the IHES, he put me in a modest Paris hotel. The two of them had me buy the book of maps *Paris par Arrondissement*, and they armed me with advice about eating, about manners and dress inside and outside of Paris, about tipping, about using the Métro, and about sightseeing. They took me for dinner one evening at the Paris restaurant Chez La Mère Michel, which had two Michelin stars at the time, and we went for lunch one day at a famous bistro named Chez Allard; Eli said that on a visit some years before, he had been eating at Chez Allard and saw Charlie Chaplin sitting at a nearby table. Elly took me shopping at two Paris department stores, so that I could buy presents for my family at home. One evening Eli and Elly took me to a piano recital in a university lecture room, the pianist being a young Vladimir Ashkenazy. Another evening they took me to a movie theater, so that I could see the system with ushers. Eli arranged that on one day Jacques Dixmier and his wife would take the three of us by train to Amiens north of Paris, where Mme. Dixmier was a professor, so that we could be given a private tour of the cathedral. On the train in one direction, I sat with Dixmier, and we had a

conversation about the algebra that Eli and I had been led to. Except for that conversation, the mathematics that Eli and I did in Paris was not memorable, but the education about living was priceless.

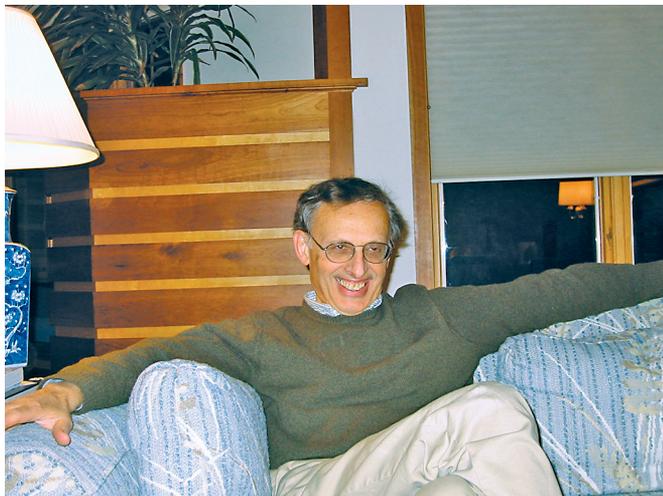
Back in the United States Eli and I tried to make progress with understanding other series of representations besides the principal series. We made little progress until we learned at the International Congress in Vancouver in August 1974 that Bill Casselman had proved a certain result known as the “subrepresentation theorem.” We quickly saw that this was the tool we needed to get started. One further discovery was needed, and this came while we were walking on a street in Salt Lake City during a conference in January 1975.

Our project was completed by early 1976, and we had published seven announcements. But publication of the details was delayed while I did a three-year stint in politics with the Ithaca public school system. Afterward we wrote a long paper explaining everything, and we got together in Ithaca once more to investigate a mysterious new kind of integral operator that was convergent only by coincidence, or so it seemed. We published remarks about that in the volume for a conference in June 1980 but were never able to handle more complicated examples.

Eli took pride in the students he had. Charles Fefferman was a student during the year I began work with Eli, and it was clear to Eli and me that the “multiplier problem for the ball,” essentially whether spherical truncations of the Fourier transform converge in  $L^p$  norm for any  $p$  other than 2, had Charlie’s name written on it. After Charlie’s remarkable thesis included an improvement in what was known about the multiplier problem, I asked Eli how he had dealt with Charlie as a thesis student. Eli explained that he first gave Charlie two warm-up problems in succession that he was fairly sure Charlie could do, and then he set him to work on Riesz summability for the multiplier problem. This approach had led to a success. A lesser advisor might have given Charlie the multiplier problem outright, leaving him to flounder. Parenthetically, Charlie a few years later gave a negative answer to the full multiplier problem.

As Eli’s reputation grew in the late 1960s and early 1970s, I became conscious that our conversations in his office were being interrupted more and more often by telephone calls from other mathematicians, perhaps five minutes per call. In fact, what he was doing was free high-level consulting, moving from one topic to another without a break. It occurred to me later that he had found an unusually efficient way to keep abreast of several areas of mathematics at once.

In the early 1970s when Eli was department chairman, we were working in his office one day when a knock came



**Figure 6.** Eli Stein in Madison, Wisconsin, 1998.

at the door, and someone came in and asked a question about representation theory. After the person left, Eli asked me, with a twinkle in his eye, “What did you think of that?” “Not bad for an instructor,” I allowed. Eli replied, “He’s a first-year graduate student.” It was Gregg Zuckerman, who became one of Eli’s PhD students and later a professor at Yale.

### *Fulvio Ricci*

The mark left by Eli Stein in the development of analysis is impressive. His intuitions and results, as well as the methods he introduced, have shaped, expanded, and revolutionized entire chapters of harmonic analysis and triggered the impressive development that this field has seen and is still going on. Much has been said about this in the *Bulletin of the American Mathematical Society* [2], and I don’t need to add anything else in this regard. I would rather say a few words about some aspects of his personality which also contributed, in my opinion, to making him so influential and charismatic.

Harmonic analysis is a very large and articulated field. It also provides important tools for many other parts of mathematics. Through his work and his more general mathematical activity, Eli has constantly promoted a unitary view of the field and cross-relations among his followers. “We have to reach out” is an expression that I like to remember, and that he used during the organizational meeting of a conference.

Eli lived through old and glorious periods of harmonic analysis. He was a graduate student in the early days of Calderón-Zygmund theory, and Littlewood was still active

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at that time. He was a source of historical details, such as stories of breakthrough ideas, connections among works by different people, controversies over priority. Such issues could also emerge during a casual conversation. His reconstructions were always rigorous and precise (as well as in conversations on politics, his other favorite topic). It is very sad for us to lose these opportunities to be in closer contact with our scientific forefathers.

Eli was a very sociable person, capable of creating a positive atmosphere around himself. I believe that this has also contributed, to some extent, to the progress of harmonic analysis.

### *Christopher Sogge*

I was very fortunate to be a student of Elias Stein during a time when he was proving a number of pioneering results that highlighted the important role that curvature plays in harmonic analysis. This theme has continued to this day and has led to a number of major results in areas such as linear and nonlinear partial differential equations, number theory, and, of course, Fourier analysis. He had an amazing ability to not only prove beautiful results, but, again and again, to formulate problems that would shape the direction of analysis for decades.

Stein influenced an amazing number of people through collaborations and mentoring. He was the PhD advisor of over fifty students, including two Fields Medalists. He was also a very generous mentor who had an enormous impact on so many of us through all stages of our careers. Personally, I learned from him the importance of striving to develop a “big picture” and tell a story, as opposed to just solving individual problems. He was able to do this in many ways, not the least of which is through the many wonderful books that he wrote that have profoundly influenced generations of undergraduates, graduate students, and researchers.

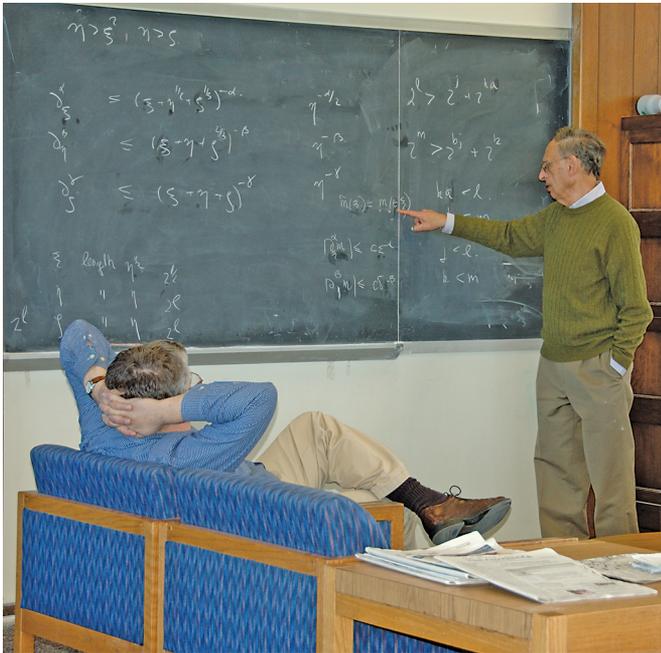
### *Duong H. Phong*

It is with great sadness and regret that I write these few lines in memory of my late teacher Elias M. Stein. To his many students, he taught not just advanced mathematics, but many lessons in life that fundamentally shaped us all. His monumental achievements and mathematical legacy have been described in detail in the article of Charles Fefferman et al. published earlier in the *Bulletin of the AMS*

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**Figure 7.** Eli Stein in discussion with Alex Nagel in Madison, Wisconsin, 2006.

[2], so I shall reminisce here only on the more personal souvenirs.

One of the greatest privileges of my life is to have collaborated with Eli throughout my career. Eli told me once that mathematicians would inevitably drift apart unless they collaborate. So even though there may be periods with very little progress or when our interests may evolve, we tried hard to keep our collaboration going. It was all to my advantage of course, but I sensed also his efforts, and I was all the more grateful to him, as he certainly did not need me. We ended up spending a lot of time together, and it was humbling for me to witness first-hand such an established figure not accept any half-truth, struggle with his own confusion at times, and strive always for correctness and clarity. I still follow his more practical advice: try and select a good note at which to end the day, so as to give oneself a better rest and better chance for the long haul. And I remember well his understated expression “I have got the feeling that. . .,” when he had more often than not seen how to solve the problem at hand. I shall forever miss this expression.

Over the years, we developed a routine of meeting alternatively at Princeton and at Columbia. Many of our meeting places have now a special meaning for me, including Princeton’s Prospect and Woodrow Wilson school cafeteria and several Italian restaurants in New York, as Eli had a predilection for Italian cuisine. We also met at many conferences. I recall conferences at Cortona, Paris, Pisa, Prague, and many others. The research that we did there,



**Figure 8.** Alexandru Ionescu, Stephen Wainger, Elias Stein, Charles Fefferman, and Duong Phong at Eli’s 80th birthday conference in Princeton, 2011.

particularly at a bistro near St Sulpice, adds another dimension to the beauty of these places. Another cherished memory is a drive with Eli and several fellow Fourier analysts to Riquewihr during a free afternoon at an Oberwolfach workshop. But perhaps most vivid are for me occasions when Eli would come to New York City, and I would get integrated in his kind and wonderful family: sometimes Eli’s wife Elly or his daughter Karen would join us for dinner, or often, he and I would go pay his mother a visit, as she lived at that time in an apartment in the Upper West Side. Even though there will be no more such occasions, their memories will stay with me always.

### *Carlos E. Kenig*

Eli Stein was an outstanding mathematician whose influence will be felt for generations to come. He was also a great teacher, mentor, and friend. I have had the immense good fortune to have personally benefited from all of these attributes. My first (and defining) close interactions with Eli took place in the period 1978–1980, when I was an instructor at Princeton. Eli was extremely busy at the time, with a large number of graduate students, post-docs, and collaborators, working on a large array of topics. In spite of this he taught masterful, crystal clear graduate classes, which I attended, on curvature in harmonic analysis, singular integrals in several complex variables, and multiparameter Hardy spaces. Eli made these very difficult topics appear deceptively simple! During this time, I looked forward to my weekly meetings with him, which were truly amazing. Given the large number of people that

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he had to meet on a regular basis, we had a fixed schedule, and we were allocated half an hour each. But what a half hour this was! I prepared for this extremely carefully, to be able to get to the heart of the matter as quickly as possible. Eli would immediately grasp the essence of the issue and invariably (no matter what the topic was) he was able to make incisive and helpful suggestions. It was truly amazing! Over the years, after I left Princeton, Eli remained a steadfast mentor and unconditional supporter, and we had many more happy interactions. He also became a friend, offering his help, advice, and personal support through difficult times.

In the last few years, we met less frequently, but I remember clearly and with great admiration, a conversation that we had the last time we met. He expressed his great love for mathematics, and his enduring desire to continue doing mathematics, which he carried out to the very end. Eli Stein will be long remembered because of his commitment and contributions to mathematics, and also, by those of us who had the opportunity to have known and loved him, because of his great human qualities.

### *Alexandru Ionescu*

Elias Stein was a brilliant mathematician and an inspiring teacher and adviser.

My initial plan going to Princeton in the fall of 1995 was to study topology. This changed however after taking a beautiful class with Eli in the following Spring semester. Together with a fellow graduate student in my year (Ken Koenig, currently at Ohio State University) we started studying analysis and we both continued on as Eli's doctoral students.

Eli was a broad mathematician, who made fundamental contributions to many areas in analysis. As an adviser, his experience and vast knowledge made all the difference. There were many instances in my graduate studies when a few minutes of talking to him significantly improved my understanding of certain ideas, as Eli knew better than anyone how to place ideas in the right framework to make them natural and clear.

Eli was a very special person and generous mathematician. He will be missed by the entire analysis community.

### *Galia Dafni*

I was very fortunate to have been a PhD student of Eli Stein's, and to have been given the chance to collaborate

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with him. His mathematical influence on me, which endures to this day in all aspects of my work, started even before I met him, during my first year at Princeton, when he was on sabbatical. I was introduced to his book, *Singular Integrals*, by one of my fellow first-year students, who had read it in Poland, in Russian, and I heard it referred to as the bible by Jean-Lin Journé. Everyone knows Stein's books: those classics and slim bright orange monographs I learned from, the big tome I refer my students to, and the recent undergraduate textbooks that initiated my daughter to analysis. Stein's courses at Princeton, which covered more advanced material and his current research interests, were as if those books had come to life. Even now, when I read the phrase "the heart of the matter" in one of his books or papers, I can still hear him saying it in that classroom in Fine Hall. I kept all my notes from those courses, but they do not do justice to the clarity, careful construction, and beauty of his lectures. Someone once told me, in a slightly accusing way, that Stein made things seem too simple. If only more people had that "defect!"

I cannot describe Eli Stein as a supervisor in general, because I only knew how he interacted with me, and to me this is what was great about his supervision: it was completely focused on the mathematics in question, not on anything or anyone else. He let me proceed at my own pace, which was quite slow, but there was no criticism, just an occasional "So?," said loudly and accompanied by a slight tilt of the head, when I met him in the elevator, to prompt me to report on my progress (or lack thereof). He always seemed to know how things were going to turn out—I recall on occasion him saying "that's the way the cookie crumbles." We continued to meet, but less frequently, after I left Princeton and we were collaborating on a paper, joint with Der-Chen Chang, on Hardy spaces on domains. I remember, on a visit to Princeton, I made the mistake of convincing Eli to avoid waiting in line at the Small World Coffee, and instead go down the street to an ice cream place where one could get an espresso for \$1. He was quite unhappy when he was handed his espresso in a paper cup.

In subsequent years we met more rarely. I recall in particular how generous he was with his time and ideas during some meetings Andrea Fraser and I had with him in Princeton. I also saw him at conferences, some of them in his honor. On every occasion he greeted me with a big smile and was interested in talking about mathematics. This was especially true in Bonn in the summer of 2014. Our last meeting was in Wrocław in 2017. I realize now, to my regret, that I never took the opportunity to say to him: thank you.

## Terence Tao

Eli was an amazingly effective advisor; throughout my graduate studies I think he never had fewer than five graduate students, and there was often a line outside his door when he was meeting with students such as myself. (The Mathematics Genealogy Project lists 52 students of Eli, but if anything this is an under-estimate.) My weekly meetings with Eli would tend to go something like this: I would report on all the many different things I had tried over the past week, without much success, to solve my current research problem; Eli would listen patiently to everything I said, concentrate for a moment, and then go over to his filing cabinet and fish out a preprint to hand to me, saying “I think the authors in this paper encountered similar problems and resolved it using Method X.” I would then go back to my office and read the preprint, and indeed they had faced something similar and I could often adapt the techniques there to resolve my immediate obstacles (only to encounter further ones for the next week, but that’s the way research tends to go, especially as a graduate student). Amongst other things, these meetings impressed upon me the value of mathematical experience, by being able to make more key progress on a problem in a handful of minutes than I was able to accomplish in a whole week. (There is a well-known story about the famous engineer Charles Steinmetz fixing a broken piece of machinery by making a chalk mark; my meetings with Eli often had a similar feel to them.)

Eli’s lectures were always masterpieces of clarity. In one hour, he would set up a theorem, motivate it, explain the strategy, and execute it flawlessly; even after twenty years of teaching my own classes, I have yet to figure out his secret of somehow always being able to arrive at the natural finale of a mathematical presentation at the end of each hour without having to improvise at least a little bit halfway during the lecture. The clear and self-contained nature of his lectures (and his many books) were a large reason why I decided to specialize as a graduate student in harmonic analysis (though I would eventually return to other interests, such as analytic number theory, many years after my graduate studies).

Looking back at my time with Eli, I now realize that he was extraordinarily patient and understanding with the brash and naive teenager he had to meet with every week. A key turning point in my own career came after my oral qualifying exams, in which I very nearly failed due to my overconfidence and lack of preparation, particularly in my

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**Figure 9.** Eli Stein with many students, collaborators, and colleagues at his 80th birthday conference in Princeton in 2011.

chosen specialty of harmonic analysis. After the exam, he sat down with me and told me, as gently and diplomatically as possible, that my performance was a disappointment, and that I seriously needed to solidify my mathematical knowledge. This turned out to be exactly what I needed to hear; I got motivated to actually work properly so as not to disappoint my advisor again.

So many of us in the field of harmonic analysis were connected to Eli in one way or another; the field always felt to me like a large extended family, with Eli as one of the patriarchs. He will be greatly missed.

## Charles Fefferman

Rather than recapitulate generalities in praise of Eli, I’d like to recall a few anecdotes, one of which I’ve adapted from an article in the *Bulletin of the AMS* [1].

Let me set the stage for my first anecdote.

In 1967, Eli returned to Princeton from a year’s leave at the University of Paris. He gave an inspiring graduate course on the topics that would later appear in his wonderful book *Singular Integrals and Differentiability Properties of Functions*. I followed that course as a second-year grad student, interested in analysis but otherwise without direction. Eli’s course was a revelation to me. The topics were simple, elegant, and fundamental. The explanations were flawless and unforgettable.

I asked Eli to be my thesis advisor. He gave me three selections from his seemingly infinite collection of interesting open problems, and I’ve never looked back.

The relevant point here is that Eli lectured with a style unlike that of any other mathematician I ever met. Many top researchers give clear, beautiful lectures; but Eli did more. Each Eli lecture began by posing a problem in the first few minutes. Eli then devoted almost the whole lecture to an explanation of how the problem fit into a bigger picture, and how to see the problem from exactly the

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right viewpoint. The last few minutes gave the complete solution. Thanks to the preparation in the preceding 45 minutes, the solution was completely obvious. After the lecture it would have taken great effort to realize that only an hour before, the listener would have had no clue how the problem might be attacked.

My first anecdote is set in the spring of 1970. The Vietnam war was raging and the Princeton campus erupted in student protest. Angry undergraduates discovered that the Princeton math department was receiving financial support from the hated Department of Defense. They demanded an explanation.

Please bear in mind that at about the same time, Princeton's Institute for Defense Analyses was surrounded by student demonstrators, and a few months later the Math Research Center at the University of Wisconsin would be destroyed by a bomb.

Eli was then serving as the chairman of the Princeton math department, and he gave a public lecture on the department's budget, in the common room of the then new Fine Hall. He presented a complete, honest, dispassionate account of the math department budget for the academic year 1969–1970. He made clear that DOD funding wasn't tied to military applications.

The students must have been satisfied, because there was no further incident at Fine Hall. I heard Eli's Lecture on the Budget with particular pleasure, because I recognized it as a quintessential Eli lecture. Eli spent almost the whole time placing the problem in context and explaining how to view it properly. The details fell into place in the last few minutes, and the math department budget was clear to all.

I hope these remarks will bring a smile to the face of anyone who has had the privilege of hearing Eli explain mathematics.

For my next story, I'll fast-forward a few years. Eli and I were working on our paper on  $H^p$  spaces. I visited Princeton from the University of Chicago, staying as a guest in Eli's home. While we were deep in thought, Eli's 11-year-old son Jeremy approached and asked "Dad, can I smoke marijuana?" Eli nonchalantly replied "When you're older," and returned to pondering the mysteries of the Lusin area integral.

Today, Jeremy is the chairman of the Harvard Economics Department, having previously served on the Board of Governors of the Federal Reserve. I don't know what use he made of his father's advice.

During that same visit, Eli and I needed a particular inequality for harmonic functions. Eli remembered reading the result in Hardy's *Collected Works*. We looked in Fine Library, but the book had been checked out. I thought to myself "checkmate," but Eli said to me, "Never mind. If

Hardy and Littlewood could prove it, so can we!" We returned to Eli's office, and an hour later we had a proof.

Optimism is a necessary condition for the hard business of proving theorems, and Eli taught optimism to me and to many others.

My final story took place about 20 years ago. I had occasion to meet with a promising Princeton undergraduate who was seriously considering dropping her study of math because she thought she wasn't smart enough. I knew that she had taken a first course in analysis from Eli, so I suggested to her that I put the question to Eli: Is this student smart enough to succeed as a mathematician? I assured the student of my great confidence in Eli's judgment.

The student sat outside my office while I put in a quick phone call to Eli. His answer was an unambiguous "YES!" The rest is history. The student stayed in mathematics, graduated as Princeton's valedictorian, and went on to do important work on the boundary between harmonic analysis and number theory. For many years, Eli was her mentor and dear friend.

That promising undergraduate was Lillian Pierce, the lead author of this article in the *Notices*.

### *Lillian B. Pierce*

In 1998 I started college at Princeton, taking the first-year course on  $\delta$ 's and  $\epsilon$ 's. I had to fly to California right before the midterm, to perform a concerto with an orchestra. I went to the professor's office, nervous and apologetic, to explain that I would miss a couple classes. The professor was unconcerned about this. Instead, he was immediately interested: Who was I? Which orchestra was I playing with? What was my favorite recording of the concerto? That was the first time I met Eli. His warmth, genuine interest, and admiration of music were unforgettable.

I took every class Eli taught during the next four years. I had an odd background before college. It would have been easy for me to fall through the cracks when I landed at Princeton. Eli, and the courses he taught, played a crucial role in making me feel at home in mathematics, and at the university.

I'll never forget a particular sequence of four undergraduate courses he taught during that time. On the first day of class, he would explain that he was going to ask us to work very hard, and that in return he would work very hard too. The lectures were beautiful, pragmatic, step-by-step explanations: Fourier analysis, complex analysis, real analysis, functional analysis. The problem sets were a full-time job, but the exams were kind; they checked we knew how to state and prove the key theorems, and then there was one elegant problem. After a couple semesters of such exams, you knew The Problem could be solved in a few lines if

you could just arrange the core results from the course correctly...

These four courses became the volumes of the *Princeton Lectures in Analysis*. The small but devoted cohort of Princeton math majors who attended the first iteration of the courses will never forget Professor Stein, or the weekly generosity of the TA's, Rami Shakarchi and Adrian Banner.

In the world of Stein mathematics, everything made sense. This was irresistible. I wrote an undergraduate thesis under his advice, then a PhD thesis, then made it as far as the IAS for a postdoc, and later came back for many visits. Gradually over two decades I realized that the unusual transparency of the math I learned from Eli was not inevitable but a consequence of work he did: thinking intensively in order to reveal the essential workings of a theorem in the simplest possible way. What would it be like to work in harmonic analysis without his perspective, and his books?

I learned to listen very carefully to everything Eli said. One sentence could hold a lesson I'd want to keep. On research: after I returned from a summer undergraduate research program, I lamented that I hadn't proved what I hoped. Eli: "But did you tell them something they didn't know?" On teaching: when I started a faculty job, Eli wanted to know what I was teaching. I started talking about definitions. Eli: "What is the first main theorem you want to reach?" On life: we sat together in my office at the IAS in 2018, and talked about his illness. Eli: "Life is a long adventure, with other adventures along the way."

Most recently, we were working on a book together. This was a true delight for me.

I was interested to find out what it took to attain Stein-quality clarity. I noticed that Eli thought it was important to explore, without worrying whether the explanation we liked at that moment might need to change again later as the final perspective emerged. Another aspect I noticed was a refusal to feel rushed. Eli conveyed the feeling that if we were doing something very well, it would not have an expiration date. I always wondered if this was another example of his characteristic kindness, because during those years I gave birth to three children, and rushing was impossible for me. In any case, I gratefully took it to heart.

Now, for most of us, it won't be possible to emulate how Eli thought and worked. He was singular.

But other practices I think we can try to sustain. His courtesy, generosity, and personal warmth. His interest in people and the world. His optimism and vigor. His encouragement to start with an example, the simplest one that captures an essential difficulty. His attitude that confusion is simply a sign that there is something interesting afoot, and an opportunity to really dig in. His insistence on knowing why a proof works, the *real* reason, which

should be simple and memorable. His patience for reaching a finished product, combined with unyielding high standards once you reach it.

I'm also going to remember how much I enjoyed talking to Eli. I'll remember the wonderful dinner parties Eli and Elly hosted, and their many kindnesses to me and my family. And sometimes I will take a walk and an espresso after lunch, just to remember Eli's great enjoyment in this tradition.

## Rami Shakarchi

Eli Stein approached me in 1998 with his bold vision of producing a series of books that would present core areas of analysis in an integrated manner and accessible to a wide audience. He wanted to emphasize the beauty and interconnections of many subjects, and take readers on a special journey. At the time, I was a first-year graduate student at Princeton University. I had finished reading Eli's book *Singular Integrals and Differentiability Properties of Functions*, and started work on my thesis with Charlie Fefferman.

My meeting with Eli was short. I accepted his offer to collaborate on the spot. His vision, ideas, and enthusiasm made for an easy decision. This was the beginning of my deep and long-lasting friendship with Eli.

Our professional collaboration lasted thirteen years during which we wrote the four-book series *Princeton Lectures in Analysis*. We quickly produced a broad outline of his vision which we agreed Eli would teach in his undergraduate classes. Eli's master plan was in motion. The classes and immediate feedback from students would make us fine tune our work and ensure that the material and its presentation would be captivating for our audience. Eli was a careful listener who judged criticism fairly. He also had an incredible gift of extracting and presenting ideas simply whilst still maintaining the right amount of technical details. Eli was fully committed to teaching the classes and the advancement of our project, which he balanced so well with his many other responsibilities.

Eli and I met every day of the week almost without fail, during the last four years that I was in graduate school. I have many fond memories of working with Eli, some of the most special ones being our lunches together. Our conversations would start as we walked to Nassau Street or one of the university dining halls, pen and notepad in hand, and pondering some particular points we were working on. Being both creatures of habit, we would often eat the same meal at the same place many days in a row, but fortunately our conversations were always varied, fun, and engaging.

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*Rami Shakarchi obtained his PhD in mathematics from Princeton University in 2002.*



**Figure 10.** Rami Shakarchi with Eli Stein at Eli's 80th birthday conference in Princeton, 2011.

Although mathematics dominated our early lunches, Eli and I developed a friendship that extended beyond our collaboration. We could talk about anything, and like a father would to his son, he gave me advice on so many aspects of life, advice that I still use and cherish to this day. A coffee (for Eli) and ice cream (for me) were *de rigeur* before returning to our offices or the Fine Hall library.

Eli and his loving wife Elly never failed to show me their friendship. On my trips back to Princeton we would always have dinner together creating wonderful memories. Eli would make me laugh so much. We could talk about a theorem one minute and about politics the next. I miss his insights, humor, and gentle nature. Eli was an incredible person who shaped my life in more ways than one. The results of our collaboration sit proudly on my bookshelf, but his support, advice, friendship, and laughter will live in my heart forever.

## Vickie Kearn

Eli Stein was the editor of the *Annals of Mathematics Studies* book series for Princeton University Press for almost 40 years. As the math editor for Princeton University Press, I had the privilege of meeting with Eli once a month for 17 of those years. We usually met in my office so Eli could browse over all of the new books (math and everything else that was newly published). He would suggest just the right referees for each manuscript we were considering for the book series and advise me on all of the great young mathematicians he knew who were emerging on the math horizon. I would chat with him about the graduate students I had met on my campus trips and why I thought it would be good to keep an eye on them as future authors. He would follow up with them and encourage them to work hard. He would also follow up with referees who were delinquent with reports. He put his heart into making this an outstanding book series for generations to come.

He was not all work and no play, however. We had wonderful discussions about politics, social injustice, and our favorite, the right-hand rule. There wasn't much we could do to solve most of the world's problems but the last topic was in our control. Quite simply, we used the traveling salesman problem and the rule of only making right-hand turns to run all of our errands.

Eli loved teaching and was very proud of his graduate students—all 52 of them! He loved to write, and along with one of his students, Rami Shakarchi, he wrote four of the most amazing textbooks that cover a wide range of topics in analysis. I often asked him about his own accomplishments but he would just smile in his magnificent way and wave his hand like they were nothing. There was one award that did touch him in a big way, I believe. He was very moved when he received the Steele Lifetime Achievement Award in 2002 for his work in analysis. He told me "This is a really big deal."

I think of him often because I picture him smiling every time I make a right-hand turn.

## Loredana Lanzani

It is sometimes the case that an exceptionally gifted scientist or artist has a limited awareness of the humanity in the people that surround them: this was not the case for Eli Stein. Among Eli's gifts was the ability to give

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**Figure 11.** Eli Stein with his mathematical family tree, in Princeton, 2011.

his undivided attention to anything his friends or collaborators needed to say: whether the chef at a Princeton crêperie expounding on the challenge of maintaining culinary integrity while making a profit, or my young self just out of graduate school trying to articulate a mathematical thought. Eli would not just listen to you: he would be absorbed in what you had to say.

I miss talking with Eli, exchanging ideas and hearing his thoughts; I miss it to an extent that I am unable to describe. But I have recently made a wonderful discovery: Eli's voice is right there in his writings. I used to read Eli's books for their mathematical content; now I also read them to hear his voice whenever I miss him most.

## Karen Stein

I've often joked that I was Eli's worst student. In fact, I was never actually his student. Like many high school kids, I needed help with my math homework and he was the obvious parent for that task. What I remember most about that frustrating—for us both—exchange was our vastly different expectations. My motivation was expediency. What teenager, I thought, wants to linger over their homework, especially if the subject is precalculus? All I wanted to

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know was which formulas I should memorize and how and when to apply them. Needless to say, Eli had different ideas. As my disinterest in the underlying principles became increasingly evident, he blurted out in exasperation that if I really understood the mathematical concepts I wouldn't need to memorize anything as I could derive the formulas on my own. Well, that brought a swift end to his tutoring; clearly, to me, he was out of his mind. Why would I need to go to all that trouble?

I now know that what I was doing was like asking someone for the last line of a novel when it's the story—in form and content—that matters, but, at the time, it made me wonder if my father understood who I was and what I cared about. Of course, later I came to understand that it was a perfect encapsulation of who he was and what he cared about—not just mathematics, but, overall, his nearly religious reverence for learning and teaching. The artist Josef Albers, another European emigrant who fled the Nazis and eventually arrived at Yale where he taught for decades, saw his mission as a teacher was “to open eyes” and the same could be said for Eli.

Like Albers, Eli had in essence two careers—as an innovator in his field (though, as has been established, I didn't study enough mathematics to truly understand in what way) and as an educational impresario. And, also like Albers, Eli was not just a guide into a hermetic world, he was the total manifestation of that world. Perhaps my disinterest in mathematics as a subject made me keenly attuned to its symbolic significance as a private language and allowed me to just observe how happy it made him to talk about mathematics with his three grandchildren. As I wasn't distracted by the content of their conversation, I could focus on the effect it had on him. That they from their early teens leaned into math, each in their own way, gave him a singular, quiet joy, though he wouldn't have said that much about it as he wouldn't have wanted them to feel any pressure from him. But it mattered to him, not because deep down he wanted them to follow in his footsteps but because mathematics had given so much to his life—shaped his life, even—and, I believe, he wanted his grandchildren to know some of its pleasures.

Over the years, Jeremy and I often marveled at how much our father knew about so many different subjects—mathematics, obviously. But also classical music, the Yankees, Shakespeare, European history, poetry, espresso, and anything and everything having to do with politics, government, and current events. And whatever he learned, whenever he had learned it, it stayed in his head as if arranged in orderly files that he could consult and refile at will. As long as I can remember it was accepted fact that he was exceptionally smart, but what was always something of a mystery about Eli, especially if you knew a bit about his



Figure 12. Eli Stein in Florence, Italy, 2004.

family history, is where did that mind come from? From what I know, his father, Elkan Stein, didn't go to college and didn't seem particularly inclined toward intellectual pursuits. He was in the diamond business, first in Antwerp, where Eli was born, and later in New York City, where the family arrived in 1941. From what I remember, my grandfather was a sweet, generous man somewhat befuddled by the interests of his middle son. At some point in my teenage years Eli explained to me that his father didn't consider him particularly successful, going on to say that my grandfather would have preferred if he had been a diamond merchant like him—a steady and reliable business, his father argued—and drive an Oldsmobile. At the time, Eli was a tenured Princeton professor who drove around in a maroon Peugeot. As little as I knew about mathematics, I considered myself worldly enough to believe my dear grandfather was sorely misguided in this, realizing then that I understood who my father was far better than my grandfather did. What struck me about how Eli conveyed all this to me is that there was no resentment on

his part—only resignation. By then, he had long accepted something he had sensed from his boyhood, his alien status in his own family, fostering a not entirely secret pride in the fact that, inwardly, he was different from his parents and brothers, entirely of his own making. What three or five year old is captivated by a spinning wheel—a machine used to polish diamonds—and fantasizes that the vision is a possible proof of perpetual motion except one that sees science as the ultimate calling? Even if that supposedly early proof proved to be a youthful illusion, his allegiance to mathematics as the path to his true self was stunningly real. His biography is the story of the flowering of his mind—a mind that forever propelled him forward, searching and questioning, until the moment it stopped.

### Jeremy Stein

I will miss my father terribly. I will miss his voice when I call on the phone, always very enthusiastic: JEREMY! But this sadness is tempered by the fact that he had a great and grand life. He most definitely did not get cheated. He lived exactly the life he wanted to live, did exactly the work he always wanted to do, and did it all with great enthusiasm and optimism.

Any long successful life has its share of good luck, and my father's life had more than its fair share. Think about it: when he's four years old, he decides he wants to be a world-famous scientist. Strange kid. But he's growing up Jewish in Hitler's Europe, which is a bit of a stumbling block. Yet somehow fortune deposits him and his family on Ellis Island in 1941. And then it's on to Stuyvesant HS, and U of Chicago—it's like the yellow brick road for somebody with his talents and dreams. It's the world of ideas just opening itself up to him and embracing him. One of the things I am most thankful for is the opportunity that this country gave him to be precisely what he was born to be, to be his best self.

Even more than that, I am most grateful for the example he set for the rest of us—by living it, not by preaching it—of how to be a certain kind of person. He was totally nonreligious and yet he had an extremely well-developed personal code. Here is what I have inferred to be his own personal set of Eli 10 commandments:

1. Do your own thing; don't be driven by money or what other people think. Work at it like a craft, not a job.
2. Be interested in everything: politics, music, movies, books, people's personal lives. Sometimes too much of the last one.

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3. Ask questions. A lot of questions. Sometimes too many.
4. Be positive, and optimistic. And spread that positivity to others.
5. Don't get into petty professional feuds. Stuff will sort itself out.
6. Share ideas and credit generously.
7. Treat everybody the same, with kindness and a genuine interest in their lives.
8. It's OK to be confident when you're good at something. Just don't let it show too much.
9. Read the *NY Times* every day.
10. Most important: do not ever find yourself forced to make a left turn in heavy traffic. Plan ahead and be prepared to take an alternate route that involves four extra miles and three right turns instead.

I love you dad, and I will miss you. But all of you will stay with me for the rest of my life. And I have Google maps now, so I'm good on the left turns. Rest easy.

**THE ELIAS M. STEIN FUND FOR THE NEXT GENERATION.** To honor Eli Stein's legacy as a mentor and teacher, Dr. Adrian Banner established the Elias M. Stein Fund for the Next Generation at the AMS. The Fund resides within the Next Generation Fund, which supports doctoral and postdoctoral mathematicians each year at modest but impactful levels, with funding for AMS programs such as travel grants, collaboration support, mentoring, and more.

In Dr. Banner's words, "Eli was passionate about helping graduate students and young mathematicians early in their careers. His wonderful teaching and textbooks have influenced thousands of budding mathematicians directly, and he genuinely cared about the mathematical and professional development of young mathematicians in general. This fund will help to continue his worthy legacy." To make a gift to the Elias M. Stein Fund for the Next Generation, please visit [www.ams.org/giving/ways-to-give/named-endowed-funds](http://www.ams.org/giving/ways-to-give/named-endowed-funds). The AMS offers donors the opportunity to establish named endowed funds to commemorate a family member, colleague, or mentor. For more information, please contact Douglas Allen, AMS Director of Development, at [dha@ams.org](mailto:dha@ams.org) or (401) 455.4126.

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