Corrections to
Advanced Real Analysis*, *Digital Second Edition

Page 42, lines 2–5 of the proof of Theorem 2.4. Change "Choose a sequence ... since K_n is simple" to "We shall choose a sequence of simple functions K_n square integrable on $X \times X$ such that each operator T_n defined by $T_n f(x) = \int_X K_n(x,y) f(y)\,d\mu(y)$ has finite-dimensional image and such that $\lim_n \|K - K_n\|_2 = 0$. The linear operator T_n is bounded with $\|T_n\| \le \|K_n\|_2$. Proposition 2.1a shows that the finite-dimensionality of the image T_n implies that T_n is compact. Since $\|T - T_n\| \le \|K - K_n\|_2$ and since the right side tends to 0, T will be exhibited as the limit of T_n in the operator norm and will be compact by Proposition 2.1b.

Page 42, after the text of line 8 of the proof of Theorem 2.4 and before the end-of-proof symbol. Insert a new paragraph as follows:
"We need to define K_n. For each integer $n \ge 1$, we can choose a measurable rectangle R_n in $X \times X$ of finite measure for which $\|K - R_n K\|_2 \le 1/n$. Application of Proposition 5.11 of *Basic* to the positive and negative parts of the real and imaginary parts of K yields a sequence $\{s_n\}$ of square integrable simple functions on $X \times X$ such that $\|s_n - K\| \le 1/n$, hence so that $\|s_n R_n - K\|_2 \le 2/n$. Say that $s_n = \sum_{k=1}^{N(n)} c_{n,k} I_{E_{n,k}}$, $c_{n,k}$ being a nonzero constant and $I_{E_{n,k}}$ being the indicator function of a subset of $R_n \subseteq X \times X$ of finite measure. We now make use of details in the proof of the Extension Theorem (Theorem 5.5 of *Basic*). Fix n. On R_n, the Extension Theorem shows how to extend the product measure $\mu \times \mu$ from the algebra of all finite disjoint unions of rectangles in R_n to the σ-algebra of measurable sets. Combining Lemma 5.32 of *Basic* with the definitions shows for each pair (n,k) that we can choose a subset $F_{n,k}$ of R_n that is in the given algebra of finite disjoint unions of rectangles within R_n and whose $\mu \times \mu$ measure is as close as we please to the measure of $E_{n,k}$. Since for fixed n, only finitely many integers k are involved, we can arrange that the simple function $K_n = \sum_{k=1}^{N(n)} c_{n,k} I_{F_{n,k}}$ has $\|K_n - t_n\|_2 \le 1/n$. Define an operator T_n using K_n in the same way that T was defined by using K. Since K_n is a simple function based on finitely many sets that are finite disjoint unions of rectangles, it has finite-dimensional image. Then the sequence $\{K_n\}$ has the required properties."

Page 51, line -8. Change "$|(Lu_i, v_i)| \le \|L\|$" to "$|(Lu_i, v_j)| \le \|L\|$".

Page 115, line 1. Change "$g = 1$" to "$g > 0$".

Page 115, line 3. Change "$g \ne 0$" to "$g = 0$".

Page 120, line -11. Change "$cp_x(f) = p_x(cf)$" to "$cp_x(f) = p_{cx}(f)$".

Page 124, line 8. Change "on" to "in".

Page 125, line -4. Change "a_{n+1}" to "t_{n+1}".

Page 128, line 6. Change "$af(x_0)$" to "$a\rho(x_0)$".

Page 129, line -2. Change "$-if(ix)$" to "$-f(ix)$".

Page 141, lines -20 and -19. Change "this linear functional takes the value" to "this linear functional takes its maximum value".

Page 141, lines -22 to -18. Change "continuous linear functional whose real part ... arrive at a contradiction" to
"continuous linear functional ℓ such that $\sup_{x \in E} \operatorname{Re} \ell(x) < \operatorname{Re} \ell(x_0)$. Let m be the maximum value of $\operatorname{Re} \ell(x)$ for x in K. The first paragraph of the proof shows that the subset of K where $\operatorname{Re} \ell$ takes the value m is a face of K, and the second part of the proof shows that this face has an extreme point x_1. Since x_1 is in E, $m = \operatorname{Re} \ell(x_1) \le \sup_{x \in E} \operatorname{Re} \ell(x) < \operatorname{Re} \ell(x_0) \le \sup_{x \in K} \operatorname{Re} \ell(x) = m$, and we arrive at the contradiction $m < m$"

Page 153, line 2 at the end. Insert footnote mark with a footnote saying, "Another way of proceeding is to use the remarks after the statement of the present corollary to identify λ as the composition of the continuous quotient mapping of $\mathcal{A}$ onto $\mathcal{A}/I$, followed by the isomorphism of the Banach space $\mathcal{A}/I$ with $\mathbb{C}$."

Page 158, line -3. Change "$\mathcal{A}$" to "$\mathcal{A}_m^*$".

Page 162, lines 7 to 10. Change "suppose that B lies in $\mathcal{A}'$ ... This proves (iv)" to "suppose that B lies in $\mathcal{A}'$. Since $\mathcal{A}^*$ is stable under $(\,\cdot\,)^*$, $B + B^*$ and $iB - iB^*$ lie in $\mathcal{A}$. Then $B + B^*$ and $\mathcal{A}$ together generate a C^* subalgbera that is commutative and contains $\mathcal{A}$. By maximality, $B + B^*$ lies in $\mathcal{A}$. Similarly $iB - iB^*$ and $\mathcal{A}$ together generate a C^* subalgebra that is commutative and contains $\mathcal{A}$. Again by maximality, $iB - iB^*$ lies in $\mathcal{A}$. Consequently the linear combination B of $B + B^*$ and $i(B - B^*)$ lies in $\mathcal{A}$. This proves (iv)".

Page 162, lines -3 and -2. Change "Since $E^* = E$ and since $\mathcal{A}$ is stable under $(\,\cdot\,)^*$," to "This conclusion applied to A^* gives".

Page 162, line -2. Change "Consequently" to "Since $E^* = E$".
Page 164, line -1. Change "$\ell(A^*A)$" to "$\ell(\widehat{A^*A})$".

Page 215, lines 8–12. Change "This descends to ... and this is open" to
"The mapping $1 \times q$ descends to a well-defined one-one continuous mapping $\widetilde{q} : (G\times G)(H\times H)/(1\times H)$ given by $(g, a)(1\times H) \mapsto (g, aH)$. In symbols, $\widetilde{q}\circ r = 1\times q$, where r is the quotient mapping $R : G\times G \to (G\times G)/(1\times H)$ given by $r(g_1, g_2) = (g_1, g_2)(1\times H)$. If U is open in $(G \times G)/(1 \times H)$, then $r^{-1}(U)$ is open in $G \times G$, and the equality of sets $\widetilde{q}(U) = \widetilde{q}(rr^{-1}(U)) = (1 \times q)(r^{-1}(U))$ shows that $\widetilde{q}(U)$ is open".

Page 216, line 9. Change "X_x" to "V_x"

Page 219, line 3 of the proof of Corollary 6.7. Change "$\sup_{x\in G} |f(gx - f(g_0x)| < \epsilon$" to "$\sup_{x\in G} |f(gx) - f(g_0x)| < \epsilon$".

Page 222, line -10 of the text. Change "$(f^{L_h})^{L_g}$" on the left end of the line to "$(f^{L_g})^{L_h}$".

Page 231, lines 7 and 10. Change "K support(f)" to "K^{-1} support(f)" at both occurrences.

Page 234, line -6 (with the display counted as one line). Change "$\widetilde{K}'$" to "$\widetilde{K}$".

Page 238, line 12. Change "Proposition 6.8" to "Proposition 6.8 of *Basic*".

Page 238, line 13. Change "$f(xy^{-1})g(y)$" to "$f(xy^{-1})h(y)$".

Page 259, line -5. Change "sum" to "algebraic direct sum".

Page 260, line 1. Change "for every N and every $u \in U$" to "for every N".

Page 262, last four lines. Change "$\chi_\tau(x)$" to "$\overline{\chi_\tau(x)}$" in three places.

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