Complex function theory and dynamics

The theory of holomorphic functions is one of the central areas of modern mathematics, and is connected to many different branches of math, including number theory, combinatorics, geometry, and dynamics.

Even simple examples of holomorphic functions can exhibit rich dynamical and geometric properties. For example, repeated compositions of a polynomial with itself can have both stable and chaotic behavior, and the places where they are chaotic often form a fractal set. Periodic points, which are fixed by some repeated composition, reveal much information about the behavior. How many periodic points there are, and how they are distributed, are interesting questions that have been extensively explored.

Another interesting property of holomorphic functions is that they preserve angles, except possibly at critical points. Many questions about how much they distort length and area, and where critical points and values are positioned, remain unanswered even for polynomials. These questions are also closely related to hyperbolic geometry, in both dimension 2 and 3.

The goal of the independent study part of this project is to explore and understand different aspects of complex functions, from analytic, geometric, and dynamical points of view. A few students will have the opportunity to continue their study by working on a research project during the summer, leading to an honors thesis. Possible research topics include counting periodic points and polynomials with prescribed properties, positions of critical points and values of polynomials, dynamics of Kleinian groups, and entire functions.