# RESEARCH STATEMENT

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## Part 1. Research summary

### 1. Every rationally connected variety over the function field of a curve has a rational point

A nonempty, smooth, projective variety over an uncountable, algebraically closed field is *rationally connected* if every pair of closed points is in the image of a regular morphism from $\mathbb{P}^1$ to the variety. A smooth, projective variety over an arbitrary field $k$ is rationally connected if its base-change to one (and hence every) uncountable, algebraically closed field is rationally connected.

**Theorem 1.1** (Graber, Harris, Starr [12]). *Every rationally connected variety $X$ defined over the function field $K$ of a curve $B$ over a characteristic $0$, algebraically closed field $k$ has a $K$-rational point. Equivalently, every projective, surjective morphism $\pi : \mathcal{X} \to B$ whose general fiber is rationally connected has an algebraic section.*

**Theorem 1.2** (de Jong, Starr [4]). *Replacing "rationally connected" by "separably rationally connected", the previous theorem holds in arbitrary characteristic.*

Theorem 1.1 was posed as a question by J. Kollár, Y. Miyaoka and S. Mori in their paper [21], and proved by them when $\dim(X)$ is 1 or 2. Also, Also, the special case of a smooth Fano hypersurface was proved by Tsen [19, Thm IV.5.4].

**Corollary 1.3.** *Let $(R, m)$ be a complete DVR containing its residue field, assumed algebraically closed. Let $\mathcal{X}$ be a regular, projective $R$-scheme. If the geometric generic fiber $\mathcal{X} \otimes_R \overline{K}(R)$ is normal and separably rationally connected, then the closed fiber $\mathcal{X} \otimes_R k$ is reduced on a nonempty open subset.*

This is the local version of Theorem 1.2 from which it follows by the Artin approximation theorem.

As another corollary of Theorem 1.2, Kollár proved that every smooth, connected, projective, separably rationally connected scheme over an algebraically closed field has trivial algebraic fundamental group [10, Cor. 3.6]. This was previously proved by Campana [1] and Debarre [9, Cor. 4.18] in the special case that $k$ has characteristic zero.

Finally, Corollary 1.6 connects two fundamental conjectures regarding uniruled and rationally connected varieties.

**Conjecture 1.4** (Hard Dichotomy Conjecture, Conj 3.3.3 [23]). *Let $char(k) = 0$ and let $X$ be a smooth projective variety. If $h^0(X, \omega_X^{\otimes n})$ equals $0$ for all positive $n$, then $X$ is uniruled.*

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**Conjecture 1.5** (Mumford's Conjecture, Conj IV.3.8.1 [19])**.** *Let char$(k) = 0$ and let $X$ be a smooth projective variety. If $h^0(X, (\Omega^1_X)^{\otimes n}) = 0$ for all positive $n$, then $X$ is rationally connected.*

**Corollary 1.6.** *Conjecture 1.4 implies Conjecture 1.5.*

## 2. Rational connectedness and sections of families over curves

Let $f: X \to B$ be a surjective morphism of projective schemes of fiber dimension $d$ such that $B$ is irreducible and normal. For every smooth curve $C \subset B$, denote by $f_C: X_C \to C$ the base-change of $f$ by $C \to B$.

**Theorem 2.1** (Graber, Harris, Mazur, Starr [11])**.** *If $f_C: X_C \to C$ has a section for every smooth curve $C \subset B$, then $X$ contains a closed subvariety $Z$ whose geometric generic fiber $Z \times_B Spec(\overline{K}(B))$ is rationally connected.*

By Theorem 1.1, if $X$ has a closed subvariety $Z$ whose geometric generic fiber is rationally connected, then every base-change $X_C \to C$ has a section. Thus Theorem 2.1 is a converse to part of Theorem 1.1.

Theorem 2.1 implied the first answer to a question posed by Serre to Grothendieck.

**Question 2.2** (Serre's Question [2])**.** Does a variety over the function field of a curve have a rational point when it is $\mathcal{O}$-acyclic, i.e., when $h^i(X, \mathcal{O}_X) = 0$ for every $i > 0$?

One can ask this question for any field. Serre was motivated by the case of a finite field, for which a positive answer was proved by Katz [18]. Nevertheless the answer to Question 2.2 is negative.

**Corollary 2.3** (Graber, Harris, Mazur, Starr [11])**.** *There exists a smooth projective curve $C$ over $\mathbb{C}$ with function field $K = K(C)$ and a smooth projective surface $X$ over $K$ such that $h^0(X, \Omega^1_X) = h^0(X, \Omega^2_X) = 0$ and such that $X$ has no $K$-rational point. In fact $X$ is a polarized Enriques surface over $K$.*

The corollary follows from Theorem 2.1 by considering the universal Enriques surface $\mathcal{X}$ over a certain parameter space $B$ of polarized Enriques surfaces. By simple properties of the Chow groups of $\mathcal{X}$, there is no subvariety $Z \subset \mathcal{X}$ as in Theorem 2.1. Therefore there is a smooth curve $C \subset B$ such that $f_C: \mathcal{X}_C \to C$ has no section. The generic fiber of $\mathcal{X}_C$ gives a negative answer to Question 2.2.

Hélène Esnault pointed out non-existence of $K$-rational points in Corollary 2.3 may possibly follow from an obstruction in the Galois cohomology of $K$. All known examples of obstructions satisfy restriction and corestriction. This means the order of the obstruction class divides the degree of $L/K$ for every residue field $L$ of a closed point of $X$. Equivalently, the order divides the degree of every $K$-rational 0-cycle on $X$. However, by explicit construction, there exist Enriques surfaces as in Corollary 2.3 having a $K$-rational 0-cycle of degree 1. Therefore non-existence of $K$-rational points is not explained by an obstruction satisfying restriction and corestriction.

**Theorem 2.4** (Starr [27])**.** *There exists a smooth projective curve $C$ over $\mathbb{C}$ with function field $K = K(C)$ and a smooth Enriques surface $X$ over $K$ with no $K$-rational point but with a $K$-rational 0-cycle of degree 1.*

## 3. Rational points of a variety over the function field of a surface

Let $k$ be an algebraically closed field of characteristic 0, let $B$ be an algebraic surface over $k$ with function field $K$, and let $X$ be a projective $K$-scheme such that $X \otimes_K \overline{K}$ is irreducible and smooth. There is an obstruction to existence of $K$-rational points on $X$ in the Brauer group of $K$. We call this obstruction a *Brauer obstruction*, though it is often also called the *elementary obstruction.*

**Theorem 3.1** (de Jong, Starr [7])**.** *If the Brauer obstruction vanishes, if $X \otimes_K \overline{K}$ is rationally connected, if for each $e$ one (hence every) projective birational model of the parameter space $RatCurves^e(X \otimes_K \overline{K})_{p,q}$ for rational curves in $X \otimes_K \overline{K}$ containing a pair of closed points $p, q$ is rationally connected, if $X \otimes_K \overline{K}$ has a* very twisting *family of pointed lines, and if specific additional hypotheses hold, then $X$ has a $K$-rational point.*

Unfortunately, the specific additional hypotheses are quite strong. We hope they can be removed, but have not yet proved this. Nonetheless, all the hypotheses do hold when $X \otimes_K \overline{K}$ is a Grassmannian or certain other homogeneous spaces. Consequently, Theorem 3.1 implies another proof of de Jong's Period-Index Theorem.

**Theorem 3.2** (de Jong [3])**.** *For every division algebra $D$ with center $K$, $dim_K(D)$ equals the square of the order of $[D]$ in the Brauer group of $K$.*

## 4. Rational curves on low degree hypersurfaces

The parameter space $\mathrm{RatCurves}^e(X)$ for degree $e$ rational curves on a projective variety $X$ figures prominently in every preceding theorem. This research project probes more deeply the structure of $\mathrm{RatCurves}^e(X)$ for the simplest rationally connected varieties: general hypersurfaces of degree $d \leq n$ in $\mathbb{P}^n$.

**Theorem 4.1** (Harris, Roth, Starr [14])**.** *If $d < \frac{n+1}{2}$, $RatCurves^e(X)$ is an irreducible, reduced, local complete intersection scheme of dimension $(n+1-d)e+(n-4)$. Moreover, it is a dense open subset of a geometrically meaningful compactification which is also a local complete intersection space.*

**Theorem 4.2** (Harris, Starr [15], [26], de Jong, Starr [8])**.** *If $d^2 \leq n+1$, every smooth, projective model of $RatCurves^e(X)$ is rationally connected, therefore has negative Kodaira dimension. For $e \geq 2$, the same holds for the space $RatCurves^e(X)_{p,q}$ parametrizing rational curves containing a fixed, but general, pair $p, q$ of points. If $d^2 \leq n$, there exists a* very twisting *family of pointed lines in $X$.*

**Theorem 4.3** (Starr [25])**.** *If $d^2 > n+1$, for every $e = \lfloor (n+1-d)/(d^2-n-1) \rfloor, \dots, n-d$, every smooth, projective model of $RatCurves^e(X)$ is of general type, i.e., the Kodaira dimension equals the usual complex dimension.*

**Theorem 4.4** (Starr [25])**.** *If $d^2 < n+1$, for every $e = 1, \dots, n-d$, $RatCurves^e(X)$ has a geometrically meaningful compactification which is a normal, $\mathbb{Q}$-Fano variety.*

**Theorem 4.5** (de Jong, Starr [6])**.** *There is an explicit, general formula for the canonical divisor class on $\overline{\mathcal{M}}_{0,0}(X, e)$ whenever it is irreducible, reduced and normal of the expected dimension.*

## 5. Hilbert and Quot functors of Deligne-Mumford stacks

Deligne-Mumford stacks occur naturally in the theory of moduli spaces and parameter spaces. This project aimed to construct analogues for Deligne-Mumford stacks of some of the parameter spaces useful in the study of schemes. Hilbert schemes and Quot schemes are very basic examples of parameter spaces, building blocks for other important spaces such as Picard schemes and moduli spaces of vector bundles and coherent sheaves.

**Theorem 5.1** (Olsson-Starr [24])**.** *Let $\mathcal{X}/S$ be a separated, locally finitely presented Deligne-Mumford stack over an algebraic space $S$, and let $\mathcal{F}$ be a locally finitely presented, quasi-coherent sheaf on $\mathcal{X}$.*

(1) *The Quot functor $Quot(\mathcal{F}/\mathcal{X}/S)$ is represented by an algebraic space separated and locally finitely presented over $S$.*
(2) *If $S$ is an affine scheme and if $\mathcal{X}$ is a tame, global quotient stack whose coarse moduli space is a quasi-projective $S$-scheme, then the connected components of $Quot(\mathcal{F}/\mathcal{X}/S)$ are quasi-projective $S$-schemes.*

# Part 2. Research proposal

## 6. Brief overview

The two main projects I am working on are:

(1) Existence of rational points of an variety defined over a non-algebraically-closed field.
(2) Properties of varieties related to rationality.

The first project aims to give some answers to the following problem.

**Problem 6.1** (Kollár, Prob 6.1.2 [19])**.** Let $F$ be a field and $X_F$ a variety over $F$. Find conditions on $F$ and $X_F$ which imply that $X_F$ has a point in $F$.

The second project concerns several suggested properties of a variety generalizing rationality in characteristic 0. Three weak generalizations are:

(1) $X$ is *ruled* if $X$ is birational to $Y \times \mathbb{P}^1$, i.e. $K(X) \cong K(Y)(t)$.
(2) $X$ is *uniruled* if there is a generically finite, dominant morphism $f : Y \times \mathbb{P}^1 \to X$, i.e. $K(Y)(t)$ is a finite extension of $K(X)$.
(3) $X$ has *negative Kodaira dimension* if $h^0(X, \omega_X^{\otimes n})$ equals 0 for all $n > 0$.

Every ruled variety is uniruled, and every uniruled variety has negative Kodaira dimension. There are uniruled varieties which are not ruled. But it is unknown whether every variety of negative Kodaira dimension is uniruled. Conjecture 1.4 states that this is true. There are several consequences of Conjecture 1.4, and one of the goals of the second project is to prove or disprove one of these consequences.

Three strong generalizations of rationality are:

(1) $X$ is *unirational* if there is a generically finite, dominant morphism $f : \mathbb{P}^n \to X$, i.e. $k(t_1, \dots, t_n)$ is a finite extension of $K(X)$.
(2) $X$ is *rationally connected* if there exists a morphism $f : \mathbb{P}^1 \to X$ whose image is contained in the smooth locus of $X$ and such that $f^*T_X$ is an ample vector bundle.
(3) $X$ satisfies *Mumford's condition* if $h^0(X, (\Omega_X^1)^{\otimes n})$ equals 0 for all $n > 0$.

Every unirational variety is rationally connected. And every rationally connected variety satisfies Mumford's condition. It has been conjectured that there are rationally connected varieties which are not unirational. And Conjecture 1.5 states that every variety satisfying Mumford's condition is rationally connected. The main goal of the second project is to investigate these conjectures.

## 7. Rational points of varieties over the function field of a surface

Let $k$ be an algebraically closed field of characteristic 0 and let $K$ be a finitely generated extension of $k$ of transcendence degree $r$. Theorem 1.1 is an answer to Problem 6.1 when $r = 1$. Theorem 3.1 is an answer to Problem 6.1 when $r = 2$. But the additional hypotheses of that theorem are unreasonably strong, and limit the usefulness of the theorem. A guiding result is the theorem of Tsen [28] and Lang [22] that a hypersurface $X$ in $\mathbb{P}^n_K$ has a $K$-rational point if the degree $d$ satisfies $d^2 \leq n$. Theorem 4.2 implies the main hypotheses of Theorem 3.1 for such a hypersurface. Unfortunately, the "additional hypotheses" of Theorem 3.1 are not satisfied. The goal is to replace these unreasonable hypotheses by reasonable hypotheses, hypotheses satisfed by the hypersurfaces in the Tsen-Lang theorem.

## 8. Rational simple-connectedness

Rational connectedness in algebraic geometry is strongly analogous to path-connectedness in topology; replace continuous maps from the unit interval by algebraic morphisms from $\mathbb{P}^1$ to go from one to the other. Following this logic, simple-connectedness in topology should have an algebraic geometry analogue which Barry Mazur introduced and calls *rational simple-connectedness*. Since simple-connectedness is path-connectedness of the space of based paths, the algebraic geometry analogue is rational connectedness of the spaces $\mathrm{RatCurves}^e(X, d)_{p,q}$ parametrizing rational curves in $X$, of fixed degree $e$, containing a pair $p, q$ of general points. This is one of the main hypotheses of Theorem 3.1. It is also the property proved for hypersurfaces in Theorem 4.2.

Unfortunately, this property is difficult to prove in general. The first hint at a general criterion for rational simple-connectedness is Theorem 4.4. By a theorem of Qi Zhang [30] and Hacon-McKernan [13], a variety with ample anticanonical bundle and Kawamata log terminal singularities is rationally connected. Assume $\mathrm{RatCurves}^e(X)_{p,q}$ is irreducible, reduced and normal of the expected dimension. Using Theorem 4.5 when both $c_1(T_X)$ and $c_1(T_X)^2 - 2c_2(T_x)$ are positive, one birational model of $\mathrm{RatCurves}^e(X)_{p,q}$ has ample anticanonical bundle; namely the contraction of the boundary in $\overline{\mathcal{M}}_{0,0}(X, e)$. Thus, to prove it is rationally connected, it suffices to prove it has Kawamata log terminal singularities. In principle, this can be proved locally using deformation theory. One goal is to analyze the singularities of this contraction, hopefully leading to a proof that it has Kawamata log terminal singularities.

Also, when $c_1(T_X)^2 - 2c_2(T_X)$ is nef, in a suitable sense, de Jong and I have a bend-and-break argument to prove $\mathrm{RatCurves}^e(X)_{p,q}$ is uniruled, without using the computation of the canonical bundle. An alternative approach is to push the bend-and-break argument further, hopefully leading to a direct proof that $\mathrm{RatCurves}^e(X)_{p,q}$ is rationally connected.

## 9. Mumford's Conjecture and the Hard Dichotomy Conjecture

The goal of this project is to investigate Conjectures 1.4 and 1.5. Both of these conjectures imply another, more accessible conjecture, Conjecture 9.2 below. First the Hard Dichotomy Conjecture needs to be rephrased. Let $k$ be an algebraically closed field with $\mathrm{char}(k) = 0$, let $X$ be a smooth, projective $k$-scheme and let $f: X \to Q$ be the *maximally rationally connected fibration* of $X$, i.e., the unique dominant rational transformation with rationally connected fibers and with $Q$ non-uniruled. For every $n > 0$ and every section $s \in H^0(X, (\Omega^1_X)^{\otimes n})$, let $\tilde{s} : T_X \to (\Omega^1_X)^{\otimes(n-1)}$ be the map of locally free sheaves defined by contracting $s$ with the given tangent vector. Define $\mathcal{F} \subset T_X$ to be the coherent subsheaf which is the intersection over all $n > 0$ and all $s$ of $\mathrm{Ker}(\tilde{s})$.

**Conjecture 9.1** (Variant of Conjecture 1.4)**.** *There exists a Zariski dense open set $U \subset X$ contained in the domain of $f$ such that $\mathcal{F}|_U$ equals the kernel of $df: T_X \to f^*T_Q$.*

If $X$ satisfies Mumford's condition, then $\mathcal{F} = T_X$ so that the maximally rationally connected fibration of $X$ is just the constant map, i.e. $X$ is rationally connected. Therefore Conjecture 9.1 implies Mumford's Conjecture. Moreover the proof of Corollary 1.6 also proves that Conjecture 9.1 is equivalent to Conjecture 1.4.

One corollary of Conjecture 9.1 would be the following result:

**Conjecture 9.2.** *There exists a dense open subset $U \subset X$ such that $\mathcal{F}|_U$ is algebraically integrable. In particular $\mathcal{F}|_U$ satisfies the Frobenius integrability condition.*

This conjecture is more accessible than Conjecture 9.1. For instance, the Frobenius integrability condition is a local rather than global condition. And Grothendieck has a conjecture predicting when an algebraic foliation is algebraically integrable. The goal of this project is to prove Conjecture 9.2, or at least reduce it to Grothendieck's conjecture, by directly determining whether $\mathcal{F}$ satisfies the Frobenius integrability condition, and whether the hypotheses of Grothendieck's conjecture hold. This would provide indirect evidence for Conjectures 1.4 and 1.5.

## 10. Unirationality and rational connectedness

This project concerns a long-standing conjecture that there exist non-unirational complex Fano manifolds. General hypersurfaces in $\mathbb{P}^n$ of degree $d \leq n$ are complex Fano manifolds. Kollár suggested a strategy in [20]: prove the Fano manifold has few rational surfaces. Equivalently, the strategy is to prove $\mathrm{RatCurves}^e(X)$ contains few rational curves. The first step is to prove $\mathrm{RatCurves}^e(X)$ is not uniruled.

**Conjecture 10.1** (Starr)**.** *Let $n \geq 6$ and let $1 \leq d \leq n-3$ be an integer such that $d < \frac{n+1}{2}$ and such that $d^2 \geq n+2$. Let $X \subset \mathbb{P}^n$ be a general hypersurface of degree $d$ and let $e > 0$ be an integer. Then every smooth, projective model of $\mathrm{RatCurves}^e(X)$ is non-uniruled. In fact it is of general type.*

The same ingredients as in Section 3 suggest $\mathrm{RatCurves}^e(X)$ is of general type: it has a projective birational model whose canonical bundle is big. Thus the conjecture reduces to proving the model has canonical singularities. As in Section 3, this problem is local in nature, and can be approached using deformation theory.

## 11. Cubic fourfolds and rational curves

Let $k$ be an algebraically closed field with $\mathrm{char}(k) = 0$. Let $X \subset \mathbb{P}^5$ be a smooth cubic hypersurface. A longstanding problem in algebraic geometry is to determine if $X$ is rational when $X$ is very general. Some cubic fourfolds are known to be rational, in particular Hassett has found new examples of rational cubic fourfolds in [17], making use of his analysis in [16] of Hodge structures of cubic fourfolds. One key tool in analyzing the Hodge structure of a cubic fourfold is the fact that $\mathrm{RatCurves}^1(X)$ is a hyperKähler manifold which is a deformation of $\mathrm{Hilb}^2(S)$ for some K3 surface $S$. Considering the usefulness of this fact, it seems reasonable to ask if $\mathrm{RatCurves}^e(X)$ might be a hyperKähler for some $e > 0$. The goal of this project is to answer this question.

Of course $\mathrm{RatCurves}^e(X)$ is not proper and may be singular. The precise question considered is:

**Question 11.1.** Let $X \subset \mathbb{P}^5$ be a cubic hypersurface which is very general. For $e \geq 5$ and odd, is $\overline{M}_{0,0}(X, e)$ birational to a hyperKähler manifold? Is $\overline{M}_{0,0}(X, e)$ isomorphic to a Hilbert scheme $\mathrm{Hilb}^f(S)$ for some K3 surface $S$?

The expectation is that the sequence of all the schemes $\mathrm{RatCurves}^e(X)$ will give an invariant of $X$, roughly the monoid of effective cycles in $\mathrm{CH}_0(S)$ in the special case that $\mathrm{RatCurves}^1(X) \cong \mathrm{Hilb}^2(S)$. The hope is to then extend this invariant to all rationally connected fourfolds, find some factor of this invariant which is birationally invariant, and use this birational invariant to prove that $X$ is irrational.

So far this project has led to the following theorem.

**Theorem 11.2** (de Jong, Starr [5])**.** *Let $X \subset \mathbb{P}^5$ be a cubic hypersurface which is very general. For every $e > 0$, the Deligne-Mumford stack $\overline{\mathcal{M}}_{0,0}(X, e)$ is an integral, local complete intersection scheme and admits a regular 2-form $\omega$.*

(1) *If $e \geq 5$ is odd, $\omega$ is nondegenerate on a nonempty open subset.*
(2) *If $e \geq 6$ is even, $\omega$ is degenerate and the associated sheaf homomorphism $T_X \to \Omega_X$ has a rank 1 kernel $\mathcal{K}$.*

The strategy for answering the first part of this question is to determine the divisor in $\overline{\mathcal{M}}_{0,0}(X, e)$ where $\omega$ is degenerate and then to determine whether this divisor can be contracted. The divisor class of $\omega$ follows from Theorem 4.5. I do not yet know if this divisor can be contracted.

Another part of this project is to prove Conjecture 9.2 for $\overline{\mathcal{M}}_{0,0}(X, e)$ when $e \geq 6$ is even. By Theorem 11.2, the sheaf $\mathcal{F}$ is either zero or it is $\mathcal{K}$. Conjecture 9.2 states that for every even $e$ either $\mathcal{F}$ is zero, or $\mathcal{K}$ is algebraically integrable and the leaves are rational curves. Using a result of Viehweg [29], to prove this statement, it suffices to prove that $\mathcal{K}$ is algebraically integrable. So far I have proved this for the case $e = 6$.

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