RESEARCH STATEMENT

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I am an algebraic geometer working on Hodge theory and its applications to the study of the geometry and topology of complex algebraic varieties. My research over the past few years has focused on degeneration problems—how the cohomology groups of a family of nonsingular algebraic varieties behave as the varieties in question degenerate to a singular one. I have applied this knowledge to the construction of extension spaces that control the degenerations, and that can be used to define numerical invariants. I am also very interested in derived categories and equivalences between them, and in the interactions between Hodge theory and derived categories that occur in several places in algebraic geometry.

The geometry of complex projective varieties is a rich and classical subject. On the one hand, such a variety X can be defined by homogeneous polynomial equations inside complex projective space, and can be studied using methods from algebraic geometry. On the other hand, when X is nonsingular, it is also a compact complex manifold with a natural Kähler metric, and can be studied using methods from differential geometry. An example of this is the Hodge decomposition

$$H^k(X, \mathbb{C}) = \bigoplus_{p+q=k} H^{p,q}(X)$$

of the cohomology groups of a complex projective manifold X; classes in the subspace $H^{p,q}(X)$ are represented by closed differential forms of type (p, q), harmonic with respect to the Kähler metric. Such a decomposition is called a Hodge structure of weight k, and *Hodge theory* is the study of Hodge structures and their interactions with the geometry of the underlying varieties.

The most important open problem in Hodge theory is the Hodge conjecture [Hod52], which predicts the existence of algebraic subvarieties of X corresponding to certain cohomology classes called Hodge classes. At its heart is the question to what extent the geometry of an algebraic variety is determined by its cohomology groups. This question is of great importance to algebraic geometry: for instance, Grothendieck's conjectural theory of motives [Gro69], which attempts to unify several subfields, depends on a positive solution to the Hodge conjecture.

The conjecture remains unproved except for the case of divisors ($k = 1$), and in a small number of very special cases [Lew99]. As I will explain below, many of the problems that I have worked on are related, directly or indirectly, to the Hodge conjecture.

Three of my results

Before going into details, I would like to summarize three results that give a good idea of the scope and nature of my work.

1. Hyperplane sections and residues. The study of hyperplane sections of smooth projective varieties is important both for an inductive approach to the Hodge conjecture, and as a testing ground for questions about more general degenerations. By work of Griffiths [Gri69] and Green [Gre85], the cohomology groups of nonsingular hyperplane sections can be computed via residues of meromorphic forms (generalizing the classical theory of residues in one complex variable); moreover, the Hodge filtration is essentially the filtration by pole order. This result has found numerous applications, such as Green's proof of the Torelli theorem for hyperplane sections [Gre85]. It is known that this nice description breaks down when the hyperplane section has singularities [DS06].

In my dissertation and in [Sch10b], I used filtered $\mathcal{D}$-modules to extend the residue calculus to the family of all hyperplane sections. I showed that the resulting filtered $\mathcal{D}$-module on the dual projective space is a minimal extension in the sense of Saito's theory of mixed Hodge modules [Sai90], and this remains one of the few examples where a concrete description of the minimal extension is possible. As an application, I found a new proof for the results of Brosnan, Fang, Nie, and Pearlstein [BFNP09] on the topology of the family of hyperplane sections. The new method also lead to a big improvement in their basic vanishing theorem, answering a question left open in their paper. (This improvement was independently discovered by Beilinson.)

2. Néron models and admissible normal functions. The Hodge conjecture is closely connected to the theory of normal functions, which are holomorphic sections of certain bundles of complex tori called intermediate Jacobians. In fact, a theorem of Zucker [Zuc76] shows that there is a one-to-one correspondence between Hodge classes and normal functions for the family of nonsingular hyperplane sections. A new approach to the Hodge conjecture, through "singularities" of normal functions, has recently been proposed by Green and Griffiths [GG07, GG06]. The singularity of a normal function is a locally defined cohomological invariant, and roughly speaking, the Hodge conjecture becomes equivalent to proving that normal functions associated to primitive Hodge classes have nontrivial singularities [BFNP09, dCM09].

To study the boundary behavior of normal functions, many people have worked on the construction of Néron models, which here means spaces that extend the bundles of intermediate Jacobians [Zuc76, Cle83, Sai96, GGK10, BPS08]. However, those constructions only worked in special cases, and the resulting spaces were poorly understood. In [Sch09a], I solved this problem by constructing Néron models for arbitrary families of intermediate Jacobians; not only that, but the new models are analytic spaces, instead of just topological spaces. The construction uses methods from Saito's theory, and is based on a new and delicate norm estimate for variations of Hodge structure of odd weight. In a subsequent joint paper with Saito [SS09], we described explicitly how the new model is related to that of [GGK10] in the one-variable case.

My construction gives geometric meaning to the cohomological notion of singularities, because an admissible normal function locally extends to a holomorphic section if and only if its singularity is zero. More importantly, I showed that the graph of any admissible normal function has an analytic closure in my model; one consequence is a more conceptual proof for the conjecture of Green and Griffiths that the zero locus of an admissible normal function on an algebraic variety is algebraic (proved shortly before by Brosnan and Pearlstein [BP09c]). This result provides the best evidence so far for the existence of the conjectural Bloch-Beilinson filtration on Chow groups, a big open problem in the study of algebraic cycles [Jan95].

3. Derived invariance of the first Betti number. One can think of the derived category of coherent sheaves on a smooth projective variety as a replacement, in algebraic geometry, for the ring of smooth functions on a manifold. Just as kernel functions give rise to integral transforms, objects in the derived category of a product $X \times Y$ give rise to so-called Fourier-Mukai transforms between the derived categories of X and Y. The most famous example is the work of Mukai [Muk81], who showed that the derived category of an abelian variety is equivalent to that of its dual. Derived categories also play an important role in birational geometry [Kaw09], for instance in the work of Bridgeland on threefold flops [Bri02], and in the mathematics used in string theory (Kontsevich's homological mirror symmetry conjecture [Kon95]).

Two varieties with isomorphic derived categories are said to be derived equivalent; they share many invariants, such as dimension, Kodaira dimension, or minimality, and it is expected [BK98] that they also have the same Hodge numbers $h^{p,q} = \dim H^{p,q}(X)$. The latter is known for curves and surfaces through the derived invariance of Hochschild cohomology.

In joint work with Popa [PS09], we proved that derived equivalent varieties have isogenous Picard varieties and automorphism groups. This implies in particular that the Hodge number $h^{1,0}$

is invariant under derived equivalences. It follows that all Hodge numbers of derived equivalent threefolds are the same, confirming the first nontrivial case of the general conjecture. We conjectured that the Picard varieties should themselves be derived equivalent, and this question in the surface case is currently the subject of a Ph.D thesis by Pham. Our method is also being used in another Ph.D thesis by Lombardi to study the behavior of the Albanese dimension under derived equivalences.

After our result, another natural question is whether being simply connected or not is a derived invariant. Here the answer is no, even for Calabi-Yau threefolds. I recently found an example of a simply connected Calabi-Yau threefold that is derived-equivalent to one with fundamental group $(\mathbb{Z}/8\mathbb{Z})^{\oplus 2}$, using a class of Calabi-Yau threefolds constructed by Gross and Popescu.

My other research work

As I wrote in the introduction, my work has mostly focused on degeneration problems, from the point of view of Hodge theory. Families of projective varieties, or morphisms between projective varieties, are one important source of degeneration problems. Given a morphism $f\colon X \to B$ between two smooth projective varieties, all fibers $X_t = f^{-1}(t)$ of the same dimension form a family of projective varieties. A general member X_t is smooth, because f is generically submersive, but there are usually singular fibers, too. The cohomology groups $H^k(X_t, \mathbb{C})$ of the smooth fibers, with their individual Hodge structures, are an example of a geometric variation of Hodge structure. There is also a theory of abstract variations of Hodge structure, and their degenerations have been studied intensively [CKS86, Kas86]. A modern framework for Hodge theory in families, based on perverse sheaves and $\mathcal{D}$-modules, has been created by Saito [Sai88, Sai90].

4. Néron models and the Green-Griffiths program. As mentioned above, my main contribution to the program of Green and Griffiths has been the construction of complex-analytic Néron models for families of intermediate Jacobians, and the resulting geometric interpretation for singularities of normal functions. To make further progress, it is necessary to understand the local behavior of an admissible normal function near a singularity. In particular, an interesting question is whether singularities are stable under birational maps. For families of hypersurfaces, this is true at boundary points corresponding to nodal hypersurfaces by [BFNP09]; on the other hand, Saito and Fakhruddin have given an abstract example of a singularity that disappears after blowing up the base manifold. I believe that classifying singularities by relating the structure of the cohomological invariant to the geometry of the graph closure inside the Néron model is a very promising direction.

In addition to the joint paper with Saito [SS09] where we explain how my Néron model is related to that of [GGK10] in the one-dimensional case, I also clarified the relationship with the topological Néron model introduced by Brosnan, Pearlstein, and Saito [BPS08]. Another interesting question, which has only been answered in dimension one, is how the complex-analytic construction is related to the log-geometric Néron model of Kato, Nakayama, and Usui [KNU10a, KNU10b].

5. Zero loci of normal functions. Green and Griffiths conjectured that the zero locus of an admissible normal function on an algebraic variety should be algebraic. There are now two proofs of this conjecture: one by Brosnan and Pearlstein [BP09c], as a culmination of a series of papers [BP09a, BP09b]; the other by myself, as one application of my work in [Sch09a]. In fact, I proved the following stronger statement: the graph of any admissible normal function has an analytic closure inside my Néron model. This follows from my norm estimate, with the help of a special case of the mixed SL(2)-orbit theorem [KNU08].

That result leads to a host of interesting questions about normal functions. How is the geometry of the graph closure related to the cohomological invariant of a normal function? For normal functions on algebraic varieties, is the graph closure again an algebraic variety? (This question is related to the work of Sommese [Som78] on images of period maps.) For normal functions coming from primitive Hodge classes, can the closure be interpreted directly in terms of the Hodge class? Can one develop an intersection theory that would assign an intersection number to two such graph closures? Some parts of these questions could be good thesis projects for graduate students.

The zero locus conjecture has the following implications for the Hodge conjecture. In [Sch10a], I gave a positive answer to a question of Pearlstein: if the normal function associated to a primitive Hodge class has a zero locus of positive dimension, then it must have a singularity at one of the points whether the closure of the zero locus meets the boundary. This means that one could, in theory, prove the Hodge conjecture by showing that the zero locus has positive dimension (to be technically accurate, provided the embedding of the variety into projective space is of sufficiently high degree). However, I also gave examples to show that already in the case of smooth projective surfaces (where the Hodge conjecture is known), there are many primitive Hodge classes whose normal functions do not have any zeros.

6. Poincaré bundles. Topologically trivial line bundles on a complex torus T are naturally parametrized by the dual complex torus $\hat{T}$. Consequently, there is a universal line bundle on $T \times \hat{T}$, the so-called Poincaré bundle; it plays an crucial role in the study of abelian varieties [Muk81, Bea83]. In the case of intermediate Jacobians, the associated principal $\mathbb{C}^*$-bundle has a Hodge-theoretic interpretation in terms of biextensions of mixed Hodge structures [Hai90]. One would like to extend the Poincaré bundle from a family of intermediate Jacobians to its Néron model, in order to define numerical invariants of admissible normal functions. In this direction, [GG06] described a Poincaré bundle on the one-parameter Néron model of [GGK10].

In recent work, I constructed a $\mathbb{C}^*$-bundle on the complex-analytic Néron model of §2, given an admissible normal function for the dual variation of Hodge structure. This solves "half" of the problem, since it produces a line bundle on one of the factors; I am optimistic that it will lead to a construction of the full Poincaré bundle. The degree of that bundle on the graph of a second admissible normal function without singularities (or on the graph closure in general) is then a numerical invariant associated to the pair.

For functions associated to primitive Hodge classes, I showed that one recovers the intersection number of the Hodge classes (a similar calculation has been done by Caibar and Clemens). Among other things, this gives another proof for my theorem that normal functions with positive-dimensional zero locus necessarily have singularities. I am currently working on constructing the full Poincaré bundle, and on generalizing the correspondence with biextensions to this setting.

7. The locus of Hodge classes. A Hodge class on a smooth projective variety is a class in the $2k$-th integral cohomology of the variety that can be represented by a differential form of type (k, k). The Hodge conjecture predicts that such classes come from algebraic cycles. Since algebraic cycles can be parametrized by algebraic varieties, one consequence of the conjecture is that in a family of varieties, the Hodge loci (the set of points where the flat translate of a given Hodge class remains a Hodge class) should be algebraic varieties. This is known to be true by the famous theorem of Cattani, Deligne, and Kaplan [CDK95]. The Hodge conjecture also makes predictions about the field of definition of Hodge loci, but not much is known about this part of the problem [Voi07].

In joint work with Brosnan and Pearlstein [BPS10], I extended the theorem of Cattani, Deligne, and Kaplan to the locus of Hodge classes in any admissible variation of mixed Hodge structure. The proof works by combining the existing result in the pure case with our recent work on zero loci of admissible normal functions. Our result has been used by Lewis in the study of arithmetically defined candidates for the Bloch-Beilinson filtration on Chow groups.

A very interesting direction is to compactify the locus of Hodge classes, and to use the compactification to get numerical invariants. My idea is to use similar methods as in the construction of the Néron model, building analytic spaces from filtrations on certain $\mathcal{D}$-modules. I have succeeded in carrying out this program for variations of Hodge structure of weight two; as I will explain in

§11 below, this has applications to the study of Noether-Lefschetz loci on Calabi-Yau threefolds, and to verifying certain numerical predictions made by physics.

8. Primitive cohomology. The Lefschetz theorems show that there is a close relationship between the cohomology groups of a smooth projective variety X and those of a smooth hyperplane section D. Except for the so-called primitive part, all the cohomology of X comes from the lower-dimensional variety D; when arguing by induction on the dimension, the key point is therefore to understand the primitive part. As a consequence of his connectedness theorem, Nori [Nor93] showed that it is isomorphic to the group cohomology of the variable part of $H^{\dim D}(D, \mathbb{Q})$, viewed as a representation of the monodromy group for the family of all smooth hyperplane sections.

In [Sch07b], I gave a new topological description of the primitive cohomology through the so-called tube mapping. This mapping produces a primitive class from an element of the monodromy group and a variable cohomology class invariant under that element, and I showed that such classes generate the entire primitive cohomology. If we consider the family of all smooth hyperplane sections, the variable cohomology groups form a local system, i.e., a covering space of the parameter space, and my theorem says that the primitive cohomology of X is naturally embedded into the first cohomology of this covering space. This means that its topology is fairly complicated, which is what we want to happen because the same covering space also contains the locus of Hodge classes.

When X is a Calabi-Yau threefold, Clemens [Cle05] has defined local potential functions on the local system whose gradients give equations for the locus of Hodge classes. As an application of [Sch07b], I proved that there is no globally defined potential function. In my opinion, this circle of ideas will be useful for the generalized Hodge conjecture for Calabi-Yau threefolds, which again relates the geometry of X to sub-Hodge structures in $H^3(X, \mathbb{Q})$. The conjecture is equivalent to proving that if there is a sub-Hodge structure of the form $H = H^{2,1} \oplus H^{1,2}$, then the locus of Hodge classes contains an abelian variety of dimension $\dim H^{2,1}$ with certain properties; on a purely topological level, my result in [Sch07b] provides evidence that this is the case. The next question to be answered is how to recover the Hodge structure on the primitive cohomology from the topology of the local system.

9. Duality for filtered $\mathcal{D}$-modules. Saito's theory of mixed Hodge modules provides a powerful and convenient framework for Hodge theory. It associates to every complex algebraic variety an abelian category of mixed Hodge modules, and morphisms of algebraic varieties give rise to pullback and pushforward functors. A mixed Hodge module on a smooth variety consists roughly of a perverse sheaf [BBD82] and a filtered $\mathcal{D}$-module $(\mathcal{M}, F)$ that correspond to each other under the Riemann-Hilbert correspondence. Generically, meaning on a dense open subset, a mixed Hodge module is just a variation of mixed Hodge structure. The associated graded $\operatorname{Gr}^F_\bullet \mathcal{M}$ for the filtration $F_\bullet \mathcal{M}$ on the $\mathcal{D}$-module defines a coherent sheaf on the cotangent bundle of the variety, which encodes in a rather subtle way how the variation of mixed Hodge structure degenerates. Except for the pushforward by a proper morphism [Sai88], it is a difficult problem to determine how the associated graded behaves under the various operations on mixed Hodge modules.

In [Sch09b], I described the associated graded of the Verdier dual of a mixed Hodge module, in terms of local cohomology (relative to the zero section of the cotangent bundle). I had originally found a special case of the formula in my thesis, after a lengthy computation using my work on residues (see §1). The resulting exact sequence reminded me of an exact sequence that occurs in the theory of local cohomology, and this observation eventually suggested the correct result for arbitrary mixed Hodge modules. As an application, I obtained formulas for ext-sheaves of the graded pieces of the Hodge filtration, which lead to necessary and sufficient conditions for the sheaves $\operatorname{Gr}^F_k \mathcal{M}$ to be locally free. In [Sch10b], I applied this theorem to show that, in the important case of the family of hyperplane sections of a smooth projective variety X, the sheaves $\operatorname{Gr}^F_k \mathcal{M}$ in the interesting range $-\dim X \leq k \leq 0$ are always reflexive. Among other things, this shows that the analytic space underlying the Néron model (see §2) is only mildly singular in that case.

10. Surfaces with big anticanonical class. One aim of the Minimal Model Program is the classification of algebraic varieties based on the behavior of the canonical line bundle. At one end of the spectrum are varieties of general type (whose canonical model has ample canonical bundle), at the other end are so-called Fano varieties with ample anticanonical bundle. On Fano varieties, and more generally on varieties with big and nef anticanonical bundle, the operations of the Minimal Model Program can be performed for arbitrary divisor classes; they are Mori dream spaces in the terminology of Hu and Keel [HK00].

On moduli spaces of stable maps, the anticanonical class is often big but not nef, and this led Chen to ask whether varieties with big anticanonical class are still Mori dream spaces. In a joint paper, we proved that this is true for rational surfaces, and noted that it fails in higher dimensions. The same theorem, however, was shortly afterwards proved by another group [TVAV09], and so we decided to leave our paper unpublished.

Two ongoing projects

To close this summary of my work, here are two of the projects that I am currently working on.

11. Noether-Lefschetz loci on Calabi-Yau threefolds. Calabi-Yau threefolds are three-dimensional projective manifolds with trivial canonical bundle; by Yau's famous theorem [Yau78], they admit canonical Ricci-flat metrics, one in each Kähler class. They have been the subject of intense study in the past 25 years, because of the role they play in string theory. Counting the number of holomorphic curves with given invariants is an important problem both in algebraic geometry and in physics, and physicists have found many amazing formulas for generating functions of such numbers, starting with [CdlOGP91].

Noether-Lefschetz loci for families of surfaces in the threefold are an interesting special case; they have expected dimension zero, and should therefore have a virtual number of points attached to them. Physicists predict that a certain generating function created from such numbers is a modular form, and a result for families of K3-surfaces has been proved by Maulik and Pandharipande [MP07]. In joint work with Maulik, we are studying this question for the family of hyperplane sections of a Calabi-Yau threefold. Using the methods I developed for compactifying Hodge loci, we are able to assign a virtual number of points to each Noether-Lefschetz locus that takes into account positive dimensional components and points at infinity. Currently, we are investigating the invariance of these numbers under deformations of the Calabi-Yau threefold, and their relationship with moduli spaces of sheaves on the threefold. The virtual numbers can also be computed from certain Hodge modules that are supported on boundary strata (obtained by deformation to the normal cone), and we are also working on the relationship between these Hodge modules and the original variation of Hodge structure. Our eventual goal is to prove the modularity of the resulting generating functions, for instance with the help of [KM90].

12. Canonical transforms for mixed Hodge modules. Together with Cautis, I am working on describing the effect of the pullback functor on the associated graded of a mixed Hodge module (see §9). Our goal is to get a satisfactory formalism for canonical transforms on the derived category that relates the transform of a complex of mixed Hodge modules to that of the corresponding complex of coherent sheaves on the cotangent bundle. This is motivated by work of Cautis, Kamnitzer, and Licata [CKL09] on derived equivalences between the cotangent bundles of Grassmanians; the results about the family of hyperplane sections described in §1 may also be seen as a special case. There is already a theory of canonical transforms for filtered $\mathcal{D}$-modules, developed by Laumon [Lau85], but the resulting formalism is unfortunately not compatible with the theory of mixed Hodge modules. It also does not give the correct result for the pullback functor, except in the non-characteristic case.

References

[BK98] S. Barannikov and M. Kontsevich, *Frobenius manifolds and formality of Lie algebras of polyvector fields*, International Mathematics Research Notices **4** (1998), 201–215.

[Bea83] A. Beauville, *Quelques remarques sur la transformation de Fourier dans l'anneau de Chow d'une variété abélienne*, Algebraic geometry (Tokyo/Kyoto, 1982), Lecture Notes in Mathematics, vol. 1016, Springer, Berlin, 1983, pp. 238–260.

[BBD82] A. A. Beĭlinson, J. Bernstein, and P. Deligne, *Faisceaux pervers*, Analysis and topology on singular spaces, I (Luminy, 1981), Astérisque, vol. 100, Société Mathématique de France, Paris, 1982, pp. 5–171.

[BO01] A. Bondal and D. Orlov, *Reconstruction of a variety from the derived category and groups of autoequivalences*, Compositio Mathematica **125** (2001), no. 3, 327–344.

[Bri02] T. Bridgeland, *Flops and derived categories*, Inventiones Mathematicae **147** (2002), no. 3, 613–632.

[BFNP09] P. Brosnan, H. Fang, Z. Nie, and G. Pearlstein, *Singularities of admissible normal functions*, Inventiones Mathematicae **177** (2009), no. 3, 599–629. With an appendix by Najmuddin Fakhruddin.

[BP09a] P. Brosnan and G. Pearlstein, *The zero locus of an admissible normal function*, Annals of Mathematics (2) **170** (2009), no. 2, 883–897.

[BP09b] ———, *Zero loci of admissible normal functions with torsion singularities*, Duke Mathematical Journal **150** (2009), no. 1, 77–100.

[BP09c] ———, *On the algebraicity of the zero locus of an admissible normal function* (2009), available at `arXiv:0910.0628`.

[BPS08] P. Brosnan, G. Pearlstein, and M. Saito, *A generalization of the Neron models of Green, Griffiths and Kerr* (2008), available at `arXiv:0809.5185v1`.

[BPS10] P. Brosnan, G. Pearlstein, and C. Schnell, *The locus of Hodge classes in an admissible variation of mixed Hodge structure*, Comptes Rendus de l'Académie Scientifique de Paris, Ser. I **348** (2010), 657660.

[CdlOGP91] P. Candelas, X. C. de la Ossa, P. S. Green, and L. Parkes, *A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory*, Nuclear Physics. B **359** (1991), no. 1, 21–74.

[CDK95] E. Cattani, P. Deligne, and A. Kaplan, *On the locus of Hodge classes*, Journal of the American Mathematical Society **8** (1995), no. 2, 483–506.

[CKS86] E. Cattani, A. Kaplan, and W. Schmid, *Degeneration of Hodge structures*, Annals of Mathematics **123** (1986), no. 3, 457–535.

[CKL09] S. Cautis, J. Kamnitzer, and A. Licata, *Coherent sheaves and categorical sl(2) actions* (2009), to appear in *Duke Mathematical Journal*, available at `arXiv:0902.1796`.

[Cle83] H. Clemens, *The Néron model for families of intermediate Jacobians acquiring "algebraic" singularities*, Institut des Hautes Études Scientifiques. Publications Mathématiques **58** (1983), 5–18.

[Cle05] ———, *Moduli schemes associated to K-trivial threefolds as gradient schemes*, Journal of Algebraic Geometry **14** (2005), no. 4, 705–739.

[dCM09] M. A. A. de Cataldo and L. Migliorini, *On singularities of primitive cohomology classes*, Proceedings of the American Mathematical Society **137** (2009), no. 11, 3593–3600.

[DS06] A. Dimca and M. Saito, *A generalization of Griffiths's theorem on rational integrals*, Duke Mathematical Journal **135** (2006), no. 2, 303–326.

[Gre85] M. L. Green, *The period map for hypersurface sections of high degree of an arbitrary variety*, Compositio Mathematica **55** (1985), no. 2, 135–156.

[GG07] M. L. Green and P. A. Griffiths, *Algebraic cycles and singularities of normal functions*, Algebraic cycles and motives (Grenoble, 2007), London Mathematical Society Lecture Note Series, vol. 343, Cambridge University Press, 2007, pp. 206–263.

[GG06] ———, *Algebraic cycles and singularities of normal functions. II*, Inspired by S. S. Chern, Nankai Tracts in Mathematics, vol. 11, 2006, pp. 179–268.

[GGK10] M. Green, P. Griffiths, and M. Kerr, *Néron models and limits of Abel-Jacobi mappings*, Compositio Mathematica **146** (2010), no. 2, 288–366.

[Gri69] P. A. Griffiths, *On the periods of certain rational integrals. I, II*, Annals of Mathematics **90** (1969), 460–495 and 496–541.

[Gro69] A. Grothendieck, *Standard conjectures on algebraic cycles*, Algebraic Geometry (International Colloquium, Tata Institute for Fundamental Research, Bombay, 1968), Oxford University Press, London, 1969, pp. 193–199.

[Hai90] R. Hain, *Biextensions and heights associated to curves of odd genus*, Duke Mathematical Journal **61** (1990), no. 3, 859–898.

[Hod52] W. V. D. Hodge, *The topological invariants of algebraic varieties*, Proceedings of the International Congress of Mathematicians, vol. 1 (Cambridge, Mass., 1950), American Mathematical Society, Providence, R. I., 1952, pp. 182–192.

[Jan95] U. Jannsen, *Mixed motives, motivic cohomology, and Ext-groups*, Proceedings of the International Congress of Mathematicians, vol. 1 (Zürich, 1994), Birkhäuser, Basel, 1995, pp. 667–679.

[Kas86] M. Kashiwara, *A study of variation of mixed Hodge structure*, Publications of the Research Institute for Mathematical Sciences **22** (1986), no. 5, 991–1024.

[KNU08] K. Kato, C. Nakayama, and S. Usui, SL(2)-*orbit theorem for degeneration of mixed Hodge structure*, Journal of Algebraic Geometry **17** (2008), no. 3, 401–479.

[KNU10a] ———, *Log intermediate Jacobians*, Japan Academy. Proceedings. Series A. Mathematical Sciences **86** (2010), no. 4, 73–78.

[KNU10b] ———, *Néron models in log Hodge theory by weak fans* (2010).

[Kaw09] Y. Kawamata, *Derived categories and birational geometry*, Algebraic geometry (Seattle, 2005), Proceedings of Symposia in Pure Mathematics, vol. 80, American Mathematical Society, Providence, RI, 2009, pp. 655–665.

[HK00] Y. Hu and S. Keel, *Mori dream spaces and GIT*, Michigan Mathematical Journal **48** (2000), 331–348.

[Kon95] M. Kontsevich, *Homological algebra of mirror symmetry*, Proceedings of the International Congress of Mathematicians, vol. 1 (Zürich, 1994), Birkhäuser, Basel, 1995, pp. 120–139.

[KM90] S. S. Kudla and J. J. Millson, *Intersection numbers of cycles on locally symmetric spaces and Fourier coefficients of holomorphic modular forms in several complex variables*, Institut des Hautes Études Scientifiques. Publications Mathématiques **71** (1990), 121–172.

[Lau85] G. Laumon, *Transformations canoniques et spécialisation pour les $\mathcal{D}$-modules filtrés*, Astérisque **130** (1985), 56–129. Differential systems and singularities (Luminy, 1983).

[Lew99] J. D. Lewis, *A survey of the Hodge conjecture*, 2nd ed., CRM Monograph Series, vol. 10, American Mathematical Society, Providence, RI, 1999.

[MP07] D. Maulik and R. Pandharipande, *Gromov-Witten theory and Noether-Lefschetz theory* (2007), available at `arXiv:0705.1653`.

[Muk81] S. Mukai, *Duality between $D(X)$ and $D(\hat{X})$ with its application to Picard sheaves*, Nagoya Mathematical Journal **81** (1981), 153–175.

[Nor93] M. V. Nori, *Algebraic cycles and Hodge-theoretic connectivity*, Inventiones Mathematicae **111** (1993), no. 2, 349–373.

[PS09] M. Popa and C. Schnell, *Derived invariance of the number of holomorphic 1-forms and vector fields* (2009), to appear in *Annales Scientifiques d l'École Normale Supérieure*, available at `arXiv:0912.4040`.

[Sai88] M. Saito, *Modules de Hodge polarisables*, Publications of the Research Institute for Mathematical Sciences **24** (1988), no. 6, 849–995.

[Sai90] ———, *Mixed Hodge modules*, Publications of the Research Institute for Mathematical Sciences **26** (1990), no. 2, 221–333.

[Sai96] ———, *Admissible normal functions*, Journal of Algebraic Geometry **5** (1996), no. 2, 235–276.

[Sai08] ———, *Hausdorff property of the Neron models of Green, Griffiths and Kerr* (2008), available at `arXiv:0803.2771v4`.

[SS09] M. Saito and C. Schnell, *A variant of Néron models over curves* (2009), to appear in *Manuscripta Mathematica*, available at `arXiv:0909.4276v1`.

[Sch07a] C. Schnell, *Canonical extensions of local systems* (2007), available at `arXiv:0710.2869`.

[Sch07b] ———, *Primitive cohomology and the tube mapping* (2007), to appear in *Mathematische Zeitschrift*, available at `arxiv:0711.3927`.

[Sch08] ———, *The boundary behavior of cohomology classes and singularities of normal functions*, Ph. D. thesis, Ohio State University, 2008. Available at `OhioLink ETD`.

[Sch09a] ———, *Complex analytic Néron models for arbitrary families of intermediate Jacobians* (2009), submitted, available at `arXiv:0910.0662`.

[Sch09b] ———, *Local duality and polarized Hodge modules* (2009), to appear in *Publications of the Research Institute for Mathematical Sciences*, available at `arXiv:0904.3480`.

[Sch10a] ———, *Two observations about normal function*, Clay Mathematics Proceedings, vol. 9, 2010, pp. 75–79.

[Sch10b] ———, *Residues and filtered $\mathcal{D}$-modules* (2010), submitted, available at `arXiv:1005.0543`.

[Som78] A. J. Sommese, *On the rationality of the period mapping*, Annali della Scuola Normale Superiore di Pisa. Classe di Scienze. Serie IV **5** (1978), no. 4, 683–717.

[TVAV09] D. Testa, A. Várilly-Alvarado, and M. Velasco, *Big rational surfaces* (2009), available at `arXiv:0901.1094`.

[Voi07] C. Voisin, *Hodge loci and absolute Hodge classes*, Compositio Mathematica **143** (2007), no. 4, 945–958.

[Yau78] S. T. Yau, *On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère equation. I*, Communications on Pure and Applied Mathematics **31** (1978), no. 3, 339–411.

[Zuc76] S. Zucker, *Generalized intermediate Jacobians and the theorem on normal functions*, Inventiones Mathematicae **33** (1976), no. 3, 185–222.