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**Mathematics Summer Program**

# Vectors Lesson

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## Goal

Understand vectors and solve applied problems because vectors are a cool and important concept of multivariable calculus, physics, and more!

## Lecture Notes

1. Vectors
   - Have magnitude and direction
   - Magnitude is not infinite, it has an initial and terminal point
   - Vectors are equal if they have the same magnitude and direction, no matter where they are placed
   - Vector $\rightarrow \vec{v}$ = <x,y>

2. Adding vectors
   - Place initial point of second vector at the terminal point of the first vector
   - Draw the resulting vector from the initial point of the first vector to the terminal point of the second vector
   - If $\vec{u}$ = $\langle x_1, y_1 \rangle$ and $\vec{v}$ = $\langle x_2, y_2 \rangle$, then $\vec{v}+\vec{u}$ = $\langle x_2+x_1, y_2+y_1 \rangle$

3. Subtracting vectors
   - Place initial point of second vector at the initial point of the first vector
   - Draw the resulting vector from the terminal point of the second vector to the terminal point of the first vector
   - If $\vec{u}$ = $\langle x_1, y_1 \rangle$ and $\vec{v}$ = $\langle x_2, y_2 \rangle$, then $\vec{v}-\vec{u}$ = $\langle x_2-x_1, y_2-y_1 \rangle$

4. Magnitude of a vector
   - $|\vec{v}|$ = vector magnitude
   - Square root the sum of the components squared
   - If $\vec{v}$ = <x,y> then $|\vec{v}| = \sqrt{(x^2 + y^2)}$

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5. Vector components
   - ~ right triangle (in $R^2$)
     - $|\vec{v}|\sin\theta = |\vec{v_y}|$
     - $|\vec{v}|\cos\theta = |\vec{v_x}|$
     - $|\vec{v_x}|\tan\theta = |\vec{v_y}|$
   - If we know the components, we can find the angle the vector is pointing towards

6. Unit vector
   - Vector with magnitude of 1
   - Vector divided by its magnitude
   - If $\vec{u} = \vec{v}/|\vec{v}|$, then $|\vec{u}| = 1$

7. Dot product
   - Multiplication of two vectors that results in a scalar product → •
   - If $\vec{u} = \langle x_1, y_1 \rangle$ and $\vec{v} = \langle x_2, y_2 \rangle$, then $\vec{v} \cdot \vec{u} = x_2 * x_1 + y_2 * y_1$
   - Follows the commutative, scalar multiplication, and distributive properties

8. Angle between two vectors
   - θ = angle between (the direction of) two vectors
   - If $\vec{u} = \langle x_1, y_1 \rangle$ and $\vec{v} = \langle x_2, y_2 \rangle$, then $\vec{v} \cdot \vec{u} = |\vec{v}| * |\vec{u}| * \cos\theta$ (check proof here)
     - Using the function arccos( ), we can solve for θ given $\vec{v}$ and $\vec{u}$

Note: All of this can be extended to $R^3$ by adding a *z* component to the vector