Stony Brook University
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Doctoral Defense Announcement

Abstract

Symplectic Involutions: from cubic fourfolds to hyperkähler manifolds

By

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Compact hyperkähler manifolds are one of the building blocks of Kähler manifolds with trivial first Chern class, but very few examples are known. One strategy for potentially producing new examples is to study the fixed locus of finite groups of symplectic birational transformations of the known examples. We classify symplectic birational involutions of hyperkähler manifolds deformation equivalent to O'Grady's 10 dimensional example. In particular, there are 6 possible involutions, classified by their action on the second cohomology.

From a cubic fourfold $V \subset \mathbb{P}^5$, one can construct a hyperkähler manifold deformation equivalent to O'Grady's 10 dimensional exceptional example, equipped with a Lagrangian fibration $X \to \mathbb{P}^5$. Three of the possible symplectic involutions can be obtained via this construction; explicitly, one starts with a cubic fourfold admitting an involution that induces a symplectic birational involution of X. We complete the classification of involutions of a cubic fourfold Hodge theoretically, which is equivalent to identifying a sublattice $A(V)_{prim} \subset H^4(V, \mathbb{Z})$ spanned by classes of surfaces contained in such a cubic. As a byproduct, we show that cubic fourfolds with involutions exhibit the full range of behavior with regards to rationality conjectures. In particular, a cubic fourfold V with an involution that fixes a plane $\mathbb{P}^2 \subset V$ point wise is rational.

This explicit geometric construction for these three symplectic involutions is a valuable tool for studying the fixed locus. We focus on a single example, and show that the fixed locus for a particular symplectic birational involution is the disjoint union of a symplectic surface, and a 6-dimensional symplectic variety. Both varieties inherit an induced Lagrangian fibration; we describe the smooth fibers.

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