Schedule of Talks

Monday, October 22

1:30pm **Aaron Naber** (MIT)
Classification of Tangent Cones and Lower Ricci Curvature

2:30pm *Break (refreshments on 2^{nd} floor of Simons Center)*

3:30pm **Simon Brendle** (Stanford University)
Minimal Tori in S^3 and Lawson's Conjecture

Tuesday, October 23

9:30am **Phillip Griffiths** (Institute for Advanced Study)
Automorphic cohomology and cycle spaces

10:30am *Break (refreshments in Math Tower SL-240)*

11:00am **John Wermer** (Brown University)
Function Algebras and Boundaries of Complex Varieties

noon *Lunch break*

2:00pm **Vincent Guedj** (Université P. Sabatier, Toulouse)
Convergence of the normalized Kähler-Ricci flow on Fano varieties

3:00pm *Break (refreshments on 2^{nd} floor of Simons Center)*

3:30pm **Eric Friedlander** (University of Southern California)
Intersection on Singular Varieties

Wednesday, October 24

9:30am **Cumrun Vafa** (Harvard University)
Feynman Graphs and Calabi-Yau Threefolds

10:30am *Break (refreshments in Math Tower SL-240)*

11:00am **I.M. Singer** (MIT)
Beyond the string genus

noon *Lunch break*

2:00pm **Jean-Pierre Bourguignon** (CNRS-IHÉS)
Recent Results on Spinors and Dirac Operators

3:00pm *Break (refreshments on 2^{nd} floor of Simons Center)*

3:30pm **Jeff Cheeger** (Courant Institute)
Volume Estimates on sets of points at which the regularity scale is small

Thursday, October 25

9:30am **Reese Harvey** (Rice University)
Perspectives on Elliptic PDE's

10:30am *Break (refreshments in Math Tower SL-240)*

11:00am **Conan Leung** (The Chinese University of Hong Kong)
Instantons in G_2 geometry

noon *Lunch break*

2:00pm **Mark Haskins** (Imperial College, London)
Recent progress in G_2 geometry

3:00pm *Break (refreshments on 2^{nd} floor of Simons Center)*

3:30pm **Gang Tian** (Princeton University)
Conic Kähler-Einstein metrics

Friday, October 26

9:30am **Claire Voisin** (Institut de Mathématiques de Jussieu)
Unramified cohomology and integral Hodge classes

10:30am *Break (refreshments in Math Tower SL-240)*

11:00am **Paulo Lima-Filho** (Texas A&M University)
Integral currents, equivariant cohomology and regulators for real varieties

noon *Lunch break*

2:00pm **Robert Bryant** (MSRI & UC Berkeley)
On the local classification of ambiKähler structures in dimension 4

3:00pm *Break (refreshments on 2^{nd} floor of Simons Center)*

3:30pm **José Figueroa-O'Farrill** (University of Edinburgh)
Supersymmetry of hyperbolic monopoles

Saturday, October 27

9:30am **Rick Schoen** (Stanford University)
Minimal surfaces as extremals of eigenvalue problems

10:30am *Break (refreshments on 2^{nd} floor of Simons Center)*

11:00am **Robert Hardt** (Rice University)
Some Homology and Cohomology Theories for a Metric Space

noon *Lunch break*

2:00pm **Sema Salur** (University of Rochester)
Calibrations in Contact and Symplectic Geometry

3:00pm *Break (refreshments on 2^{nd} floor of Simons Center)*

3:30pm **Alice Chang** (Princeton University)
On a class of non-local conformal invariants on asymptotic hyperbolic manifolds

Sunday, October 28

9:00am **Spiro Karigiannis** (University of Waterloo)
A survey of results about G_2 conifolds

10:15am **Nigel Hitchin** (Oxford University)
The Dirac operator for Higgs bundles

11:30am **Misha Verbitsky** (National Research University, Moscow)
Global Torelli theorem for hyperkähler manifolds

Abstracts of Lectures

listed alphabetically by speaker

Jean-Pierre Bourguignon: *Recent Results on Spinors and Dirac Operators*

Sunday, October 28. 10:15am

Spinors and Dirac Operators play for almost a century a central role in Physics. It took more time for them to play a similar role in Mathematics.

The purpose of the lecture is to review recent results in the context of using spinors and Dirac operators as tools in Riemannian geometry, emphasizing the dependance of these objects on the metric.

Results related to harmonic and more general special spinors will be in particular highlighted.

Simon Brendle: *Minimal Tori in S^3 and Lawson's Conjecture*

Monday, October 22. 4:00pm

The study of minimal surfaces is one of the oldest pursuits in differential geometry. Of particular interest is the case when the ambient manifold has constant curvature. For example, in 1966, Almgren showed that any immersed minimal surface in S^3 of genus 0 is totally geodesic, hence congruent to the equator. In 1970, Blaine Lawson discovered a large class of embedded minimal surfaces in S^3, which have genus greater than 1; he also constructed examples of minimal tori, which are immersed but fail to be embedded. Motivated by these results, Lawson conjectured that the Clifford torus is the only embedded minimal surface in S^3 of genus 1.
In this talk, I will describe a proof of this conjecture. The proof involves an application of the maximum principle to a function that depends on a pair of points on the surface.

Robert Bryant: *On the local classification of ambiKähler structures in dimension 4*

Friday, October 26. 2:00pm

An *ambiKähler structure* on a 4-manifold M is a triple $\big([g], J_+, J_-\big)$ where $[g]$ is a conformal structure on M and J_+ and J_- are $[g]$-compatible complex structures that commute, induce opposite orientations on M, and each are Kähler with respect to some Riemannian metric in the conformal class $[g]$.

These structures first arose in connection with generalized geometry and in the construction of certain σ-models in physics. They have been the subject of investigations by Apostolov and Gualtieri (arXiv:math/0605.342) and Apostolov, Calderbank, and Gauduchon (arXiv:math/1010.0992), who classified the examples with a 2-dimensional Lagrangian symmetry group and investigated their relationship with 4-dimensional Einstein orbifolds.

In this talk, after discussing the basic local structure theory, I will provide a complete local classification of these structures by interpreting the integrability conditions as an overdetermined system of PDE that can be treated by the methods of exterior differential systems. In particular, I will provide a complete list of local normal forms, except for a (somewhat mysterious) 6-dimensional family whose existence has been proved but for which the structure equations have (so far) resisted integration.

Alice Chang: *On a class of non-local conformal invariants on asymptotic hyperbolic manifolds*

Saturday, October 27. 3:30pm

We will discuss properties of a class of conformal invariants in conformal geometry and their connection to geometric quantities on asymptotically hyperbolic manifolds. Special emphasize will be on the extension theorem of Caffarelli-Silvestre and applications in this setting.

Jeff Cheeger: *Volume Estimates on sets of points at which the regularity scale is small*
Wednesday, October 24. 3:30pm

We discuss joint work with Aaron Naber in which Hausdorff dimension estimates on singular sets for certain elliptic and parabolic equations are improved to volume estimates on the set of points at which the "regularity scale" is small. For instance, for Einstein manifolds, the regularity scale at x is the "curvature radius", $r_{|Rm|}(x)$ i.e. the supremum of those $r \leq 1$, such that $\sup_{B_r(x)} |Rm| \leq r^{-2}$. Here Rm denotes the curvature tensor. After briefly indicating the scope of the applications to date, we illustrate the method by giving additional details in the Einstein case (which is typical). The key point is an effective replacement for the iterated blow up arguments used in proving the earlier Hausorff dimension estimates. This replacement enables one to work instead on a single scale.

José Figueroa-O'Farrill: *Supersymmetry of hyperbolic monopoles*
Friday, October 26. 3:30pm

Hyperbolic monopoles are solutions of the Bogomol'nyi equations on three-dimensional hyperbolic space. These equations are a natural reduction of the self-duality equations for Yang-Mills fields in four-dimensional euclidean space. After some introductory remarks on supersymmetry for mathematicians, I will present the construction of a supersymmetric Yang-Mills theory on hyperbolic space, identify hyperbolic monopoles as supersymmetric configurations and will show how supersymmetry determines the geometry of the moduli space of hyperbolic monopoles.

Eric Friedlander: *Intersection on Singular Varieties*
Tuesday, October 23. 3:30pm

Chow's Moving Lemma justifies the intersection product on rational equivalences classes of algebraic cycles on smooth varieties. This talk will discuss work in progress with Joe Ross to define an intersection product on algebraic cycles on singular (complex) varieties. Our techniques include the "moving lemma for families" of algebraic cycles which Blaine and I proved many years ago.

Phillip Griffiths: *Automorphic cohomology and cycle spaces*
Tuesday, October 23. 9:30am

Automorphic cohomology arose from Hodge theory in the late 1960's. Fairly soon thereafter, many of its representation-theoretic properties were understood. However, the geometric and arithmetic aspects of automorphic cohomology remained largely mysterious. Due in significant part to the work of Carayol, this has begun to change. In this talk we will explain some of these developments which show that automorphic cohomology exhibits a rich geometric structure in which cycles on flag domains play an important role.

Vincent Guedj: *Convergence of the normalized Kähler-Ricci flow on Fano varieties*
Tuesday, October 23. 2:00pm

Let X be a Fano manifold whose Mabuchi functional is proper. A deep result of Perelman-Tian-Zhu asserts that the normalized Kähler-Ricci flow, starting from an arbitrary Kähler form in c_1(X), smoothly converges towards the unique Kähler-Einstein metric. We will explain an alternative proof of a weaker convergence result which applies to the broader context of (log)-Fano varieties. This is joint work with Berman, Boucksom, Eyssidieux and Zeriahi.

Robert Hardt: *Some Homology and Cohomology Theories for a Metric Space*

Saturday, October 27. 11:00am

Various classes of chains and cochains may reveal geometric as well as topological properties of metric spaces. In 1957, Whitney introduced a geometric "flat norm" on polyhedral chains in Euclidean space, completed to get flat chains, and defined flat cochains as the dual space. Federer and Fleming also considered these in the sixties and seventies, for homology and cohomology of Euclidean Lipschitz neighborhood retracts. These include smooth manifolds and polyhedra, but not algebraic varieties or subspaces of some Banach spaces. In works with Thierry De Pauw and Washek Pfeffer, we find generalizations and alternate topologies for flat chains and cochains in general metric spaces. With these, we homologically characterize Lipschitz path connectedness and obtain several facts about spaces that satisfy local linear isoperimetric inequalities.

Mark Haskins: *Recent progress in G_2 geometry*

Sunday, October 28. 10:15am

In their foundational paper Calibrated Geometries, Reese and Blaine discovered two very rich calibrated geometries in 7-dimensional Euclidean space: associative 3-folds and coassociative 4-folds. Both calibrations are intimately linked with the compact exceptional Lie group G_2; in particular they exist on any Riemannian manifold with holonomy group contained in G_2. Finding compact associative 3-folds in compact manifolds with G_2 holonomy has been a particular challenge, in part because their deformation theory is less well-behaved than coassociative 4-folds. We describe recent work joint with Corti, Nordstrom and Pacini in which we construct a plentiful supply of compact G_2 manifolds that contained rigid associative 3-folds and some of the other advances we made in the process.

Reese Harvey: *Perspectives on Elliptic PDE's*

Thursday, October 25. 09:30am

Nigel Hitchen: *The Dirac operator for Higgs bundles*

Sunday, October 28. 10:15am

Spiro Karigiannis: *A survey of results about G_2 conifolds*

Sunday, October 28. 9:00am

The exceptional properties of the octonion algebra allow us to define the notion of a G_2 structure on an oriented spin 7-manifold, which is a certain "nondegenerate" 3-form that induces a Riemannian metric in a nonlinear way. The manifold is called a G_2 manifold if the 3-form is parallel. Such manifolds are always Ricci-flat, and are of interest in physics. More recently, however, there has been interest in G_2 *conifolds*, which have a finite number of isolated "cone-like" singularities.

After some background on G_2 manifolds and their moduli, we will present (an admittedly biased) survey of some results on G_2 conifolds, and the closely related asymptotically conical G_2 manifolds, including:

- a theorem (K., 2009) on desingularization of G_2 conifolds by glueing, and its implications about the G_2 moduli space
- a new result (K.-Lotay, 2012) on the deformation theory of G_2 conifolds, including several important applications
- a brief discussion about a potential method of constructing G_2 conifolds, generalizing an existing construction of smooth compact G_2 manifolds (K.-Joyce, 2013)

Paulo Lima-Filho: *Integral currents, equivariant cohomology and regulators for real varieties*
Friday, October 26. 11:00am

We provide an explicit formula - in the level of complexes - for the regulator map from the motivic cohomology of real varieties to the integral Deligne cohomology for real varieties, introduced in joint work with dosSantos. The construction requires a formulation of ordinary RO(G)-graded equivariant cohomology using complexes of real analytic currents, and some properties of Milnor K-theory sheaves. Explicit examples are constructed for Voevodsky's complexes that parallel Totaro's construction for Bloch's higher Chow groups, whose non-triviality is detected using our regulator maps.

Conan Leung: *Instantons in G_2 geometry*
Thursday, October 25. 11:00am

M-theory on G_2 manifolds is an analog of string theory on symplectic manifolds. The role of holomorphic curves with Lagrangian boundary conditions is replaced by associative submanifolds with coassociative boundary conditions. The work of Fukaya-Oh related holomorphic disks in cotangent bundles with Morse flow lines in Lagrangian submanifolds. Wang, Zhu and I generalized this to the G_2 setting, namely thin associative submanifolds can be constructed from regular holomorphic curves in coassiciative submanifolds. This can be used to construct new examples of associative submanifolds.

Aaron Naber: *Classification of Tangent Cones and Lower Ricci Curvature*
Monday, October 22. 1:00pm

We consider limit spaces $(M_i, g_i, p_i) \to (X, d, p)$, where the spaces M_i are noncollapsed and have Ricci curvature uniformly boundedfrom below. In this case we study the set $TC(p)$ of metric spaces which consists of the possible tangent cones at p, and give a classification result which says exactly which subsets of all metric spaces can arise as $TC(p)$ for some such limit. We use this to build new examples of limit spaces with particularly degenerate behaviors. In particular we show limit spaces cannot be stratified based on their tangent cones, and that there exists a limit space for which there are even nonhomeomorphic tangent cones at a point. This is joint work with Toby Colding.

Sema Salur: *Calibrations in Contact and Symplectic Geometry*
Saturday, October 27. 2:00pm

In a celebrated paper published in 1982, F. Reese Harvey and Blaine Lawson introduced four types of calibrated geometries. Special Lagrangian submanifolds of Calabi-Yau manifolds, associative and coassociative submanifolds of G_2 manifolds and Cayley submanifolds of Spin(7) manifolds. Calibrated geometries have been of growing interest over the past few years and represent one of the most mysterious classes of minimal submanifolds.

In this talk, I will first give brief introductions to G_2 manifolds, and then discuss relations between G_2 and contact structures.

If time permits, I will also show that techniques from symplectic geometry can be adapted to the G_2 setting. These are joint projects with Hyunjoo Cho, Firat Arikan and Albert Todd.

Rick Schoen: *Minimal surfaces as extremals of eigenvalue problems*

Saturday, October 27. 9:30am

For closed surfaces and for surfaces with boundary there are natural eigenvalue extremal problems whose solutions determine minimal surfaces in the sphere or the ball with a natural boundary condition. We will discuss the geometric properties of extremal metrics and the difficult problem of existence and regularity. This is joint work with A. Fraser.

I. M. Singer: *Beyond the string genus*

Wednesday, October 24. 11:00am

Gang Tian: *Conic Kähler-Einstein metrics*

Thursday, October 25. 3:30pm

Misha Verbitsky: *Global Torelli theorem for hyperkähler manifolds*

Sunday, October 28. 11:30am

A mapping class group of an oriented manifold is a quotient of its diffeomorphism group by the isotopies. We compute a mapping class group of a hypekähler manifold M, showing that it is commensurable to an arithmetic subgroup in $SO(3, b_2 - 3)$. A Teichmuller space of M is a space of complex structures on M up to isotopies. We define a birational Teichmuller space by identifying certain points corresponding to bimeromorphically equivalent manifolds, and show that the period map gives an isomorphism of the birational Teichmuller space and the corresponding period space $SO(b_2-3, 3)/SO(2) \times SO(b_2-3, 1)$. We use this result to obtain a Torelli theorem identifying any connected component of birational moduli space with a quotient of a period space by an arithmetic subgroup. When M is a Hilbert scheme of n points on a K3 surface, with $n-1$ a prime power, our Torelli theorem implies the usual Hodge-theoretic birational Torelli theorem (for other examples of hyperkähler manifolds the Hodge-theoretic Torelli theorem is known to be false).

Cumrun Vafa: *Feynman Graphs and Calabi-Yau Threefolds*

Wednesday, October 24. 9:30am

I discuss how singularities of toric Calabi-Yau threefolds relate to 5 dimensional superconformal theories. Each such singularity is captured by a Feynman-like diagram with cubic vertices. The evaluation of the diagram includes integration over the complexified Kähler moduli of Calabi-Yau and leads to the computation of the index of the resulting 5d superconformal theory.

Claire Voisin: *Unramified cohomology and integral Hodge classes*

Friday, October 26. 9:30am

Unramified cohomology of a complex algebraic variety produces important birational invariants coming from the comparison between the Zariski and Euclidean topologies and the associated Leray spectral sequence, which is the Bloch-Ogus spectral sequence. The talk will be an introduction to this subject, and we will show eventually how the non-triviality of unramified cohomology is related to the defect of the Hodge conjecture for integral Hodge classes, in adequate degrees (joint work with J.-L. Colliot-Thelene).

John Wermer: *Function Algebras and Boundaries of Complex Varieties*

Tuesday, October 23. 11:00am

Let M be a smooth, compact, oriented manifold in C^n, $dim\ M = 2p - 1$.

Question 1: Under what conditions on M does there exist a complex analytic variety V such that M is the boundary of V? Suppose such a V exists. Define A to be the algebra of smooth functions on M such that f admits a holomorphic extension to V, and let $\bar{A}$ be the uniform closure of A on M. Then $\bar{A}$ is a closed subalgebra of $C(M)$.

Question 2: Describe the elements of $\bar{A}$.

In the 1950's the case p = 1 (M is a closed curve) was thoroughly investigated. The answer for Question 1 was given by Reese Harvey and Blaine Lawson in their fundamental paper "On boundaries of complex analytic varieties,I", Ann. of Math. 102 (1975). In my talk, I shall discuss these matters.