How do teachers expect students to represent mathematical work?

A study of teachers’ recognition of routine ways that proofs are presented and checked in high school geometry

Justin Dimmel
University of Michigan

Work presented here is supported by Rackham Merit Fellowship and by NSF grants REC-0133619 and DRL-0918425. Opinions expressed here are the author’s and do not reflect the views of the University of Michigan or the National Science Foundation.
Motivation

The aim of my research is to investigate how people develop discipline-specific communication skills.

Mathematical experts use a range of semiotic resources that include speaking, writing, drawing, and gesturing when communicating about mathematics.

What opportunities are available to mathematical novices to develop proficiency with these modes of communication?
Overview of study

I conducted a two-part, descriptive study of US high school geometry to identify the opportunities that students have to develop discipline-specific communication skills in mathematics classrooms.
Overview of study

I conducted a two-part, descriptive study of US high school geometry to identify the opportunities that students have to develop discipline-specific communication skills in mathematics classrooms.

Part I: Analyzed video episodes of geometry classrooms to identify routine ways that students communicate when doing proofs.
Overview of study

I conducted a two-part, descriptive study of US high school geometry to identify the opportunities that students have to develop discipline-specific communication skills in mathematics classrooms.

Part I: Analyzed video episodes of geometry classrooms to identify routine ways that students communicate when doing proofs.

Part II: Conducted an experiment to gauge the extent to which the routines identified in Part I are recognized by secondary mathematics teachers.
Research questions

How do students communicate when doing proofs in geometry?

To what extent do secondary teachers recognize routine (i.e., normative) ways that students communicate when doing proofs?
Background (I)

The TIMSS study\(^1\) identified mathematics *teaching scripts* that describe what regularly happens in classrooms.

- general descriptions of the activity that takes place in classrooms
- valid from an external observer’s perspective
- distilled from representative samples of classroom videos

---

The TIMSS study\textsuperscript{1} identified mathematics \textit{teaching scripts} that describe what regularly happens in classrooms.

- general descriptions of the activity that takes place in classrooms
- valid from an external observer’s perspective
- distilled from representative samples of classroom videos

How could we generate more specific descriptions?

Background (I)

The TIMSS study\(^1\) identified mathematics *teaching scripts* that describe what regularly happens in classrooms.

- general descriptions of the activity that takes place in classrooms
- valid from an external observer’s perspective
- distilled from representative samples of classroom videos

How could we generate more specific descriptions?

How could we determine if something observed (by researchers) to regularly happen is expected (by teachers) to actually happen?

---

Background (I)

The TIMSS study\(^1\) identified mathematics *teaching scripts* that describe what regularly happens in classrooms.

- general descriptions of the activity that takes place in classrooms
- valid from an external observer’s perspective
- distilled from representative samples of classroom videos

How could we generate more specific descriptions?

How could we determine if something observed (by researchers) to regularly happen is expected (by teachers) to actually happen?

How could we describe specific teaching routines without analyzing a representative sample of classroom videos?

The TIMSS study\textsuperscript{1} identified mathematics \textit{teaching scripts} that describe what regularly happens in classrooms.

- general descriptions of the activity that takes place in classrooms
- valid from an external observer’s perspective
- distilled from representative samples of classroom videos

How could we generate more specific descriptions?

→ describe what occurs in specific instructional situations

How could we determine if something observed (by researchers) to regularly happen is expected (by teachers) to actually happen?

How could we describe specific teaching routines without analyzing a representative sample of classroom videos?

Background (I)

The TIMSS study\(^1\) identified mathematics *teaching scripts* that describe what regularly happens in classrooms.

- general descriptions of the activity that takes place in classrooms
- valid from an external observer’s perspective
- distilled from representative samples of classroom videos

How could we generate more specific descriptions?

→ *describe what occurs in specific instructional situations*

How could we determine if something observed (by researchers) to regularly happen is expected (by teachers) to actually happen?

→ *investigate whether teachers react to departures from regularities*

How could we describe specific teaching routines without analyzing a representative sample of classroom videos?

Background (I)

The TIMSS study\(^1\) identified mathematics *teaching scripts* that describe what regularly happens in classrooms.
- general descriptions of the activity that takes place in classrooms
- valid from an external observer’s perspective
- distilled from representative samples of classroom videos

How could we generate more specific descriptions?
→ *describe what occurs in specific instructional situations*

How could we determine if something observed (by researchers) to regularly happen is expected (by teachers) to actually happen?
→ *investigate whether teachers react to departures from regularities*

How could we describe specific teaching routines without analyzing a representative sample of classroom videos?
→ *use multimedia questionnaires*

First part of study

How do students communicate when doing proofs in geometry?
Theoretical Framework (I): Instructional Situations

Instructional situations (Herbst & Chazan, 2012) facilitate the exchange of work (produced by students) for knowledge claims (from the teacher).

- The research conducted for this study focuses on the situation of doing proofs in geometry (Herbst & Brach, 2006).

The exchange of work-for-knowledge claims is an exchange of semiotic resources.
Theoretical Framework (II): Semiotic Norms

A norm is not only something that regularly happens, but is also something that is expected (by participants in situations) to happen (Garfinkel, 1963; Herbst & Chazan, 2003)

• Norms are shared, social, and generally unnoticed unless they are breached.

• By semiotic norm, I refer to the expected ways of using semiotic resources to exchange work in instructional situations.
Conjecturing Semiotic Norms

• I reviewed video episodes of geometry classrooms doing proofs; episodes were recorded from the classes of 3 geometry teachers.

• Across the episodes, I observed recurring ways that semiotic resources were used during the activities of presenting and checking proofs.
The Sequence Norm

When students present proofs at the board, students *can* act as *transcribers*:

- the order in which the parts of a proof are transcribed need not be mathematically coherent

- students can reproduce the different parts of the argument in whatever order makes the transcription convenient
The Details Norm

For a proof to be considered acceptable, specific details (but not others) need to be included in the written statements that make up the proof:

- Students are expected to include statements that unpack explicit givens.

- Students are *not* expected to include statements that are tacitly warranted by diagrams.
Second part of study

To what extent do secondary teachers recognize routine (i.e., normative) ways that students communicate when doing proofs?
Method

• The method of investigation was a virtual breaching experiment with control.

• Variant of classic breaching experiments (Garfinkel, 1963; Rafalovich, 2006)
  - Breaches are virtual: participants view representations of situations (rather than experience breaches directly)
  - Participants are randomly assigned to treatment or control conditions
Method (II)

I used a customizable graphic language (Herbst, Chen, Chieu, & Weiss, 2012) to create storyboard probes in breach/control pairs

- Storyboards were created in breach/control pairs to represent episodes of geometry instruction

- Breach/control pairs were identical except during a 3-5 frame segment during which a teacher breaches or does not breach the norm.
Method (II)

Breach Storyboard

Segment of Interest (teacher breaches hypothesized norm)

Distracter segment

Control Storyboard

Segment of Interest (teacher complies with hypothesized norm)

Distracter segment

Participants in Group A

Participants in Group B
Developing storyboards

• Cyclical use of records of practice\textsuperscript{2}
• 4 storyboards were developed to target the \textit{details} norm.
• The storyboards breached the \textit{details} norm in distinct ways

\textit{less details} storyboards: teacher accepts a proof that omits a hypothesized-to-be-required detail

\textit{more details} storyboards: teacher insists on details that are hypothesized to be excessive

2. Jacobs, Kawanaka, & Stigler, 1999
Breach: teacher omits written detail that is hypothesized to be needed
Breach: teacher insists on detail that is hypothesized to be excessive
Design & Data

• 73 secondary mathematics teachers participated in the study.
  – Multimedia survey delivered online
• Planned comparison study; participants randomly assigned to conditions.
  - 5 experiment groups; ~ 15 participants per group.
• Each participant viewed 2 breach storyboards, 2 control storyboards.
• Participants were asked the same set of open- and closed-ended questions:
Design & Data (II)

- Open-ended question: What did you see happening in this scenario?
- Closed-ended questions
  - *Episode Appropriateness* (EA): How appropriate was the teacher’s review of the proof?
  - Two *segment appropriateness* rating questions: How appropriate were the teacher’s actions in this segment of the story?
    - *Segment of Interest* (SI): The segment where the teacher breaches/does not breach the target norm
    - *Distracter Segment* (SD): A segment of the story common across breach/control conditions
Design & Data (III)

• Participants used the same 6-valued Likert-like format to respond to each appropriateness question:

1 = Very Inappropriate
2 = Inappropriate
3 = Somewhat Inappropriate
4 = Somewhat Appropriate
5 = Appropriate
6 = Very Appropriate
Hypotheses

I. Episode appropriateness ratings (EA) for breach storyboards will be less than EA for control storyboards.
Hypotheses

I. Episode appropriateness ratings (EA) for breach storyboards will be less than EA for control storyboards

II. Segment of interest ratings for breach storyboards ($S_{IA_{breach}}$) will be less than segment of interest ratings for control storyboards ($S_{IA_{control}}$)
I. Episode appropriateness ratings (EA) for breach storyboards will be less than EA for control storyboards

II. Segment of interest ratings for breach storyboards ($S/A_{breach}$) will be less than segment of interest ratings for control storyboards ($S/A_{control}$)

III. There will be no significant difference in ratings across conditions for the distracter segment rating questions.
Hypotheses

I. Episode appropriateness ratings (EA) for breach storyboards will be less than EA for control storyboards across condition (different participants).

II. Segment of interest ratings for breach storyboards ($S_{A_{breach}}$) will be less than segment of interest ratings for control storyboards ($S_{A_{control}}$) across condition (different participants).

III. There will be no significant difference in ratings across conditions for the distracter segment rating questions.
Hypotheses

I. Episode appropriateness ratings (EA) for breach storyboards will be less than EA for control storyboards.

II. Segment of interest ratings for breach storyboards ($S_I A_{\text{breach}}$) will be less than segment of interest ratings for control storyboards ($S_I A_{\text{control}}$).

III. There will be no significant difference in ratings across conditions for the distracter segment rating questions.
Hypotheses

I. Episode appropriateness ratings (EA) for breach storyboards will be less than EA for control storyboards

II. Segment of interest ratings for breach storyboards ($S_{IA_{breach}}$) will be less than segment of interest ratings for control storyboards ($S_{IA_{control}}$)

III. There will be no significant difference in ratings across conditions for the distracter segment rating questions.

IV. For breach storyboards, $S_{IA}$ ratings will be significantly less than $S_{DA}$ ratings.

V. For control storyboards, there will be no significant difference between $S_{IA}$ ratings and $S_{DA}$ ratings.
Hypotheses (II)

Breach Storyboard

1. $EA_{(breach)} < EA_{(control)}$

Control Storyboard
Hypotheses (II)

1. \( EA_{(breach)} < EA_{(control)} \)

2. \( S/A_{(breach)} < S/A_{(control)} \)

Breach Storyboard

Segment of Interest (teacher breaches hypothesized norm)

Control Storyboard

Segment of Interest (teacher complies with hypothesized norm)
Hypotheses (II)

I. \( EA_{(breach)} < EA_{(control)} \)

II. \( S_{IA_{(breach)}} < S_{IA_{(control)}} \)

III. \( S_{DA_{(breach)}} = S_{DA_{(control)}} \)
Hypotheses (II)

Breach Storyboard

I. \(E_A\) (breach) < \(E_A\) (control)

II. \(S_I A\) (breach) < \(S_I A\) (control)

III. \(S_D A\) (breach) = \(S_D A\) (control)

IV. \(S_I A\) < \(S_D A\)

Segment of Interest (teacher breaches hypothesized norm)

Distracter segment

Control Storyboard

Segment of Interest (teacher complies with hypothesized norm)

Distracter segment
Hypotheses (II)

I. \( E_A^{(\text{breach})} < E_A^{(\text{control})} \)

II. \( S_I A^{(\text{breach})} < S_I A^{(\text{control})} \)

III. \( S_D A^{(\text{breach})} = S_D A^{(\text{control})} \)

IV. \( S_I A < S_D A \)

V. \( S_I A = S_D A \)
Results
Hypotheses I, II, III

*Less Details* Stories (BG1 vs. CG2)

<table>
<thead>
<tr>
<th>Rating</th>
<th>Breach</th>
<th>Control</th>
<th>$\mu_1 - \mu_2$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EA</td>
<td>3.4</td>
<td>4.6</td>
<td>-1.2</td>
<td>.001</td>
</tr>
<tr>
<td>$S_I A$</td>
<td>2.3</td>
<td>4.5</td>
<td>-2.2</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>$S_D A$</td>
<td>4.2</td>
<td>4.5</td>
<td>-0.3</td>
<td>.374</td>
</tr>
</tbody>
</table>

*More Details* Stories (BG2 vs. CG3)

<table>
<thead>
<tr>
<th>Rating</th>
<th>Breach</th>
<th>Control</th>
<th>$\mu_1 - \mu_2$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EA</td>
<td>3.4</td>
<td>4.4</td>
<td>-1.0</td>
<td>.006</td>
</tr>
<tr>
<td>$S_I A$</td>
<td>3.3</td>
<td>4.2</td>
<td>-0.9</td>
<td>.009</td>
</tr>
<tr>
<td>$S_D A$</td>
<td>3.8</td>
<td>4.3</td>
<td>-0.5</td>
<td>.115</td>
</tr>
</tbody>
</table>
## Hypotheses IV, V

<table>
<thead>
<tr>
<th>Group</th>
<th>Story</th>
<th>S\textsubscript{A}</th>
<th>S\textsubscript{DA}</th>
<th>(\mu_1 - \mu_2)</th>
<th>(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Breach</td>
<td>2.3</td>
<td>4.2</td>
<td>-1.9</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>n=16</td>
<td>Control</td>
<td>4.8</td>
<td>4.9</td>
<td>-0.1</td>
<td>.704</td>
</tr>
<tr>
<td>2</td>
<td>Breach</td>
<td>3.3</td>
<td>3.9</td>
<td>-0.6</td>
<td>.024</td>
</tr>
<tr>
<td>n=13</td>
<td>Control</td>
<td>4.4</td>
<td>4.5</td>
<td>-0.1</td>
<td>.819</td>
</tr>
<tr>
<td>3</td>
<td>Breach</td>
<td>2.8</td>
<td>4.4</td>
<td>-1.6</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>n=15</td>
<td>Control</td>
<td>4.2</td>
<td>4.4</td>
<td>-0.2</td>
<td>.499</td>
</tr>
</tbody>
</table>
Example Open Responses

*Less Details* Breach:

- “A proof shouldn't leave *any piece* up to the imagination or interpretation”

- “When you do proofs, *you can’t assume anything*”
Example Open Responses

**Less Details Breach:**
- “A proof shouldn't leave **any piece** up to the imagination or interpretation”
- “When you do proofs, you can’t assume anything”

**More Details Breach**
- “A 99 step proof? This teacher is **being a bit ridiculous on the thoroughness** of the proof”
- “The teacher got caught up in some minutia of the problem and is getting overly detailed about steps that really are not relevant in the solving of the problem.”
- “We assume the steps he wants to add [when] given the picture”
Discussion

• The differences in ratings suggest that:
  – Teachers recognize normative ways of checking the details of a proof
  – The details of a proof get checked in mathematically specific ways

• The details norm presents a curiosity:
  – Participants objected to the less details breaches, on the grounds that every step in a proof needs to be explicitly stated
  – Participants objected to the more details breaches, on the grounds that the teacher was seeking too much detail in the proof.
Significance

Research that describes what occurs in mathematics classrooms can:
Significance

Research that describes what occurs in mathematics classrooms can:

• generate basic knowledge about mathematics teaching
Significance

Research that describes what occurs in mathematics classrooms can:

• generate basic knowledge about mathematics teaching

• help reform efforts contend with the paradox of “change without difference” (Woodbury & Gess-Newsome, 2002),
Significance

Research that describes what occurs in mathematics classrooms can:

• generate basic knowledge about mathematics teaching

• help reform efforts contend with the paradox of “change without difference” (Woodbury & Gess-Newsome, 2002), by
Significance

Research that describes what occurs in mathematics classrooms can:

• generate basic knowledge about mathematics teaching

• help reform efforts contend with the paradox of “change without difference” (Woodbury & Gess-Newsome, 2002), by

• providing a foundation for designing instructional interventions that are aligned with the realities and constraints of existing practice (Cobb, Zhao, & Dean, 2009; Hora & Ferrare, 2013)
Implications for Research

This study of geometry instruction:

• shows that instruction can be described without analyzing a representative sample of classroom video

• shows the viability of using an experimental design to investigate the aspects of instruction that are salient for teachers
Implications for Teaching

The results of the study suggest that

• secondary geometry teachers have a shared sense of how proofs get presented and checked in geometry classrooms

• there are opportunities in geometry classrooms for students to develop discipline-specific communication skills.