MAT 132 HW 41-42

1. Problems

1. Find the Maclaurin series of the following function. And find the degree two taylor polynomial centered at zero.
$$f(x) = (1-x)^{\frac{1}{5}}$$
2. Find the Maclaurin series of the following function. And find the degree two taylor polynomial centered at zero.
$$f(x) = (1-x^2)^{-\frac{1}{5}}$$
3. Find the Maclaurin series of the following function. And find the degree two taylor polynomial centered at zero.
$$f(x) = (1-3x)^{\frac{2}{5}}$$
4. Compute
$$\lim_{x\to 0} \frac{\cos x - 1}{2x^2}.$$
5. Compute
$$\lim_{x\to 0} \frac{\tan(x) - x}{13x^3}.$$

2. Answer Key

1. Maclaurin series is
$$(1-x)^{\frac{1}{5}} = \sum_{n=0}^{\infty} \binom{\frac{1}{5}}{n} (-x)^n$$
and
$$T_2(x) = 1 - \frac{1}{5}x + \frac{-2}{25}x^2.$$
2. Maclaurin series is
$$(1-x^2)^{\frac{-1}{5}} = \sum_{n=0}^{\infty} \binom{\frac{-1}{5}}{n} (-x^2)^n$$
and
$$T_2(x) = 1 + \frac{1}{5}x^2 + \frac{2}{25}x^4.$$
3. Maclaurin series is
$$(1-2x)^{\frac{2}{5}} = \sum_{n=0}^{\infty} \binom{\frac{2}{5}}{n} (-2x)^n$$
and
$$T_2(x) = 1 + \frac{-4}{5}x + \frac{-3}{25}x^2.$$
4. $\frac{-1}{4}$.

5. $\frac{-1}{39}$.

3. Solutions

1. Using the formula:

$$(1+x)^r = \sum_{n=0}^{\infty} \binom{r}{n} x^n.$$

We see that the Maclaurin series is

$$(1-x)^{\frac{1}{5}} = \sum_{n=0}^{\infty} \binom{\frac{1}{5}}{n} (-x)^n$$

and

$$T_2(x) = 1 - \frac{1}{5}x + \frac{-2}{25}x^2.$$

2. Using the formula:

$$(1+x)^r = \sum_{n=0}^{\infty} \binom{r}{n} x^n.$$

We see that the Maclaurin series is

$$(1-x^2)^{\frac{-1}{5}} = \sum_{n=0}^{\infty} \binom{\frac{-1}{5}}{n} (-x^2)^n$$

and

$$T_2(x) = 1 + \frac{1}{5}x^2 + \frac{2}{25}x^4.$$

3. Using the formula:

$$(1+x)^r = \sum_{n=0}^{\infty} \binom{r}{n} x^n.$$

We see that the Maclaurin series is

$$(1-2x)^{\frac{2}{5}} = \sum_{n=0}^{\infty} \binom{\frac{2}{5}}{n} (-2x)^n$$

and

$$T_2(x) = 1 + \frac{-4}{5}x + \frac{-3}{25}x^2.$$

4. Taylor expand $\cos x - 1$ about 0 to get

$$\frac{-x^2}{2} + \frac{x^4}{24} + O(x^8).$$

Now divide this by $2x^2$ to get

$$\frac{-1}{4} + \frac{x^2}{48} + O(x^6).$$

Thus,

$$\lim_{x\to 0} \frac{\cos x - 1}{2x^2} = \lim_{x\to 0} \frac{-1}{4} + \frac{x^2}{48} + O(x^6) = \frac{-1}{4}.$$

5. Taylor expand $\tan x - x$ about 0 to get
$$\frac{-x^3}{3} + \frac{2x^5}{15} + O(x^9).$$
Now divide this by $13x^3$ to get
$$\frac{-1}{39} + \frac{2x^2}{195} + O(x^6).$$
Thus,
$$\lim_{x\to 0} \frac{\tan(x) - x}{13x^3} = \lim_{x\to 0} \frac{-1}{39} + \frac{2x^2}{195} + O(x^6) = \frac{-1}{39}.$$