MAT 132 HW 34-37

1. Problems

1. Consider the following power series and find radius of convergence.
$$\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$$
2. Consider the following power series and find radius of convergence.
$$\sum_{n=1}^{\infty} \frac{n^2 x^n}{3^n}$$
3. Consider the following power series and find radius of convergence.
$$\sum_{n=1}^{\infty} 3\frac{n!}{n^n} x^n$$
4. Consider the following series.
$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \qquad g(x) = \sum_{n=0}^{\infty} \frac{x^n}{2^n}.$$
   Find $f(x) + g(x)$.
5. Consider the following series.
$$f(x) = \sum_{n=0}^{\infty} \frac{3x^n}{(n+1)\cdot n!}, \qquad g(x) = \sum_{n=0}^{\infty} \frac{x^n}{3^n}.$$
   Find $2f(x) + 3g(x)$.

2. Answer Key

1. 1.
2. $\frac{1}{3}$.
3. e.
4. $\sum_{n=0}^{\infty} \left(\frac{1}{n!} + \frac{1}{2^n}\right) x^n$
5. $\sum_{n=0}^{\infty} \left(\frac{6}{(n+1)\cdot n!} + \frac{1}{3^{n-1}}\right) x^n$

3. Solutions

1.
$$\lim_{n\to\infty} \left| \frac{\frac{(3x)^{n+1}}{n+1}}{\frac{(3x)^n}{n}} \right| = \lim_{n\to\infty} \frac{3|x| \cdot n}{n+1} = |x|.$$
   So $|x| < 1$ And so the radius of convergence is 1.

2.
$$\lim_{n\to\infty}\left|\frac{\frac{(n+1)^2}{3^(n+1)}}{\frac{n^2}{3^n}}\right| = \lim_{n\to\infty}\frac{3(n+1)^2}{n^2} = 3.$$

So $|x| < \frac{1}{3}$ And so the radius of convergence is $\frac{1}{3}$.

3.
$$\lim_{n\to\infty}\left|\frac{3\frac{(n+1)!}{(n+1)^{n+1}}}{3\frac{n!}{n^n}}\right| = \lim_{n\to\infty}(n+1)\cdot\frac{n^n}{(n+1)^n\cdot(n+1)} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \frac{1}{e}.$$

So $|x| < e$ And so the radius of convergence is e.

4. Add the nth coefficient of each of the series.
$$f(x)+g(x) = \sum_{n=0}^{\infty}\left(\frac{1}{n!}+\frac{1}{2^n}\right)x^n.$$

So by the Ratio Test this converges.

5.
$$2f(x)+3g(x) = 2\sum_{n=0}^{\infty}\frac{3x^n}{(n+1)\cdot n!} + 3\sum_{n=0}^{\infty}\frac{x^n}{3^n} = \sum_{n=0}^{\infty}\left(\frac{6}{(n+1)\cdot n!}+\frac{1}{3^{n-1}}\right)x^n$$