MAT 132 HW 32-33

1. Problems

1. Consider the following series and apply the ratio test.

$$\sum_{n=1}^{\infty} \frac{(3^n n!)^3}{(3^n)^{3n}}$$

2. Consider the following series and apply the root test.

$$\sum_{n=1}^{\infty} \frac{n}{3^n}$$

3. Determine if the following series converges absolutely, conditionally, or not at all.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+2}.$$

4. Determine if the following series converges absolutely, conditionally, or not at all.

$$\sum_{n=5}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}-2}.$$

5. Determine if the following series converges absolutely, conditionally, or not at all.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(n)}{n^5}.$$

2. Answer Key

1. Converges.
2. Does not converge.
3. Does not converge.
4. Conditionally Converges.
5. Converges Absolutely.

3. Solutions

1. Consider

$$\lim_{n\to\infty} \left| \frac{\frac{(3^{n+1}(n+1)!)^3}{(3^(n+1))^{3(n+1)}}}{\frac{(3^n n!)^3}{(3^n)^{3n}}} \right| = \lim_{n\to\infty} \frac{(3(n+1))^3}{3^{(6n+3)}} = 0.$$

So by the Ratio Test this converges.

2. Consider the following limit

$$\lim_{n\to\infty} \left(\frac{n}{3^n}\right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^{\bar{n}}}{2} = \frac{1}{2}.$$

Thus, by the root test this series converges.

3. Consider:
$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+2}.$$
We note that
$$\lim n \to \infty(-1)^{n+1}\frac{n}{n+2} \neq 0.$$
Thus, the series diverges.
4. Consider:
$$\sum_{n=5}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}-2}.$$
We see that $\frac{1}{\sqrt{n}-2}$ is decreasing and
$$\lim_{n\to\infty}\frac{1}{\sqrt{n}-2} = 0.$$
So by the alternating series test this series converges. Now consider
$$\sum_{n=5}^{\infty}\frac{1}{\sqrt{n}-2}.$$
We note that $\frac{1}{\sqrt{n}} \leq \frac{1}{\sqrt{n}-2}$. And by the p-test we have that $\sum_{n=5}^{\infty}\frac{1}{\sqrt{n}}$. diverges so by comparison we that $\sum_{n=5}^{\infty}\frac{1}{\sqrt{n}-2}$ diverges. Thus,
$$\sum_{n=5}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}-2}.$$
converges conditionally.
5. Consider:
$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(n)}{n^5}.$$
Note that $|(-1)^{n+1}\sin(n)| \leq 1$ so $\frac{|(-1)^{n+1}\sin(n)|}{n^5} \leq \frac{1}{n^5}$. And by the p-test we know that $\sum_{n=1}^{\infty}\frac{1}{n^5}$ converges so by comparison we have $\sum_{n=1}^{\infty}\frac{|(-1)^{n+1}\sin(n)|}{n^5}$ converges. Thus,
$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(n)}{n^5}$$
converges absolutely.