MAT 132 HW 29-31

1. Problem

1. Does the following sequence converge or diverge? If it converge give the what it converges too.
$$\sum_{n=1}^{\infty} \frac{17}{n}.$$
2. Does the following sequence converge or diverge? If it converge give the what it converges too.
$$\sum_{n=0}^{\infty} 7^n.$$
3. Determine if the following series converge.
$$\sum_{n=3}^{\infty} \frac{1}{2n\ln(n)}.$$
4. Determine if the following series converges.
$$\sum_{n=1}^{\infty} \left(\frac{\ln(n)}{n}\right)^4.$$
5. Determine if the following series converge.
$$\sum_{n=3}^{\infty} \frac{\cos(n)+2}{n^3}.$$

2. Answer Key

1. Diverges
2. Diverges
3. Diverges.
4. Converges.
5. Converges.

3. Solution

1. $\sum_{n=1}^{\infty} \frac{17}{n} = 17 \cdot \sum_{n=1}^{\infty} \frac{1}{n}$ and we identify the harmonic series which is known to diverge.
2. $\sum_{n=0}^{\infty} 7^n$ is a geometric series and $|7| > 1$ so it diverges.
3. We will use the integral test since $f(x) = \frac{1}{2x\ln(x)}$ on $[3, \infty)$ satisfies the hypotheses of this test. So consider (We will the u-substitution $u = \ln(x)$ so $du = \frac{dx}{x}$).
$$\int_3^{\infty} \frac{1}{2x\ln(x)} dx = \frac{1}{2}\int_3^{\infty} \frac{1}{\ln(x)}\frac{dx}{x} = \frac{1}{2}\int_{\ln(3)}^{\infty} \frac{1}{u} du = \frac{1}{2}\lim_{N\to\infty} \ln(N) - \ln(\ln(3)).$$
Thus we see that this integral diverges so the sum diverges.

4. Consider
$$\sum_{n=1}^{\infty}\left(\frac{\ln(n)}{n}\right)^4.$$
We will do a limit comparison with $\frac{1}{n^2}$. So compute
$$\lim_{n\to\infty}\frac{\left(\frac{\ln(n)}{n}\right)^4}{\frac{1}{n^2}} = \lim_{n\to\infty}\frac{(\ln(n))^4}{n^2} \overset{\star}{=} 4\lim_{n\to\infty}\frac{\ln(n)}{n^2} \overset{\star}{=} 4\lim_{n\to\infty}\frac{1}{2n^2} = 0$$
Where at $\star$ we applied L'Hôpital's rule. And by limit comparison we have
$$\sum_{n=1}^{\infty}\left(\frac{\ln(n)}{n}\right)^4$$
converges.

5. Consider:
$$\sum_{n=3}^{\infty}\frac{\cos(n)+2}{n^3}.$$
Note that $\cos(n)+2 \leq 3$ so $\frac{\cos(n)+2}{n^3} \leq \frac{3}{n^3}$. And by the p-test we know that $\sum_{n=3}^{\infty}\frac{1}{n^3}$ converges so by comparison we have $\sum_{n=3}^{\infty}\frac{\cos(n)+2}{n^3}$ converges.