MAT 132 HW 26-28

1. Problems

1. Suppose $\lim_{n\to\infty} a_n = 1$ and $\lim_{n\to\infty} b_n = -1$. Find
$$\lim_{n\to\infty} (5a_n - 8b_n).$$
2. Determine if the sequence
$$a_n = \frac{2}{3^n}, n \geq 1$$
is bounded and whether it is eventually monotone, increasing, or decreasing.
3. Determine if the sequence
$$a_n = \frac{5^n}{n!}, n \geq 1.$$
converges and if it does find the limit.
4. Find the limit of the sequence:
$$\lim_{n\to\infty} \frac{17n^3 + n^2 + 100}{13n^3 + 6n^2 + n + 3}.$$
5. Find the limit of the sequence:
$$\lim_{n\to 0} e^{-n} + \frac{1}{1+n}.$$

2. Answer Key

1. 13.
2. Bounded. Eventually decreasing.
3. Convergent. Limit is 0.
4. $\frac{17}{13}$
5. 2.

3. Solutions

1. $\lim_{n\to\infty}(5a_n - 8b_n) = 5\lim_{n\to\infty} a_n - 8\lim_{n\to\infty} b_n = 5 + 8 = 13.$
2. Since for all $n \geq 1$ we have that $0 \geq 2 \leq 3^n$, we see that $0 \leq a_n \leq 1$. So the sequence is bounded. Now consider
$$\frac{a_{n+1}}{a_n} = \frac{2 \cdot 3^n}{2 \cdot 3^{n+1}} = \frac{1}{3} < 1.$$
Thus, $a_{n+1} < a_n$.
3.
$$a_{n+1} = \frac{5^{n+1}}{(n+1)!} = \frac{5}{n+1} \cdot \frac{5^n}{n!} = \frac{5}{n+1} \cdot a_n.$$

Thus, a_n is decreasing when $n \geq 4$. And we note $a_n \geq 0$ for all n. Thus, a_n is convergent and call the limit L. Moreover,

$$a_{n+1} = \frac{5}{n+1} \cdot a_n.$$

So

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{5}{n+1} \cdot a_n.$$

Thus,

$$L = 0 \cdot L.$$

And we conclude $L = 0$.

4. We notice that

$$\lim_{n\to\infty} \frac{17n^3 + n^2 + 100}{13n^3 + 6n^2 + n + 3}.$$

is the limit of a rational function and the highest power in the numerator and denominator is 3. Thus, the limit is the ratio of the coefficients of the highest power. Thus,

$$\lim_{n\to\infty} \frac{17n^3 + n^2 + 100}{13n^3 + 6n^2 + n + 3} = \frac{17}{13}.$$

5.

$$\lim_{n\to 0} e^{-n} + \frac{1}{1+n} = \lim_{n\to 0} e^{-n} + \lim_{n\to 0} \frac{1}{1+n} = e^0 + \frac{1}{1+0} = 2.$$