Homework

1. Compute the general solution to the differential equation $y' = x^2y - 2y$.

2. Compute the general solution and particular solution to the differential equation $y' = x^3e^y$ with initial condition $y(0) = 1$.

3. Compute the general solution to the differential equation $y' = 2y(\sec^2 x - 1)$.

4. Compute the general solution to the differential equation $yxy' = (x^2 - 1)$.

5. Draw a slope field for all integer coordinates $(x, y)$ with $-3 \leq x, y \leq 3$ for the following differential equations:

   (a) $y' = y - x$

   (b) $y' = 3y + xy$

Solutions

1. Compute the general solution to the differential equation $y' = x^2y - 2y$.

   Solution: We can factor the right side of the equation to get $y' = y(x^2-2)$. Expressing $y'$ as $dy/dx$, when we separate variables we get $\int \frac{dy}{y} = \int x^2 - 2dx$. Integrating this, we obtain $\ln|y| = x^3/3 - 2x + C_1$. Exponentiating, we get $|y| = e^{x^3/3-2x+C_1}$. We can bring the constant $C_1$ as the constant $C = e^{C_1}$ and this lets us remove the absolute value bars around $y$. So we have $y = Ce^{x^3/3-2x}$.

2. Compute the general solution and particular solution to the differential equation $y' = x^3e^y$ with initial condition $y(0) = 1$.

   Solution: Rewriting this equation with $dy/dx$ notation, we have $dy/dx = x^3e^y$. Separating variables, we get $\frac{dy}{e^y} = x^3dx$. To continue, we want to take the integral of both sides, as follows:

$$\begin{aligned}
&\int e^{-y}dy = \int x^3dx \\
&-e^{-y} = x^4/4 + C_1 \\
&e^{-y} = -x^4/4 + C_2 \\
&-y = \ln(-x^4/4 + C_2) \\
&y = \ln(-x^4/4 + C_2)
\end{aligned}$$

   Plugging in the initial condition $y(0) = 1$, we have $1 = \ln(C_2)$. Hence, $C_2 = 0$. Thus, the general solution is $y = \ln(-x^4/4+C)$ and the particular solution is $y = \ln(-x^4/4)$.

3. Compute the general solution to the differential equation $y' = 2y(\sec^2 x - 1)$.

   Solution: Rewriting this equation with $dy/dx$ notation, we have $dy/dx = 2y(\sec^2 x - 1)$. Separating variables, we get $\frac{dy}{y} = 2(sec^2x - 1)dx$. The computation then follows as:

$$\begin{aligned}
\int \frac{dy}{y} &= 2\int \sec^2 x - 1dx \\
\ln|y| &= 2(\tan x - x + C_1) \\
|y| &= e^{2(\tan x - x + C_1)} \\
y &= Ce^{2\tan x - 2x}
\end{aligned}$$

   And we are done.

4. Compute the general solution to the differential equation $yxy' = (x^2 - 1)$.

Solution: Rewriting this equation with $dy/dx$ notation, we have $yx\frac{dy}{dx} = (x^2 - 1)$. Separating variables, we get $ydy = \dfrac{x^2 - 1}{x}dx$. The computation then follows as:

$$\int ydy = \int x - (1/x)dx$$
$$y^2/2 = x^2/2 - \ln|x| + C_1$$
$$y^2 = x^2 - 2\ln|x| + C$$

And we are done.

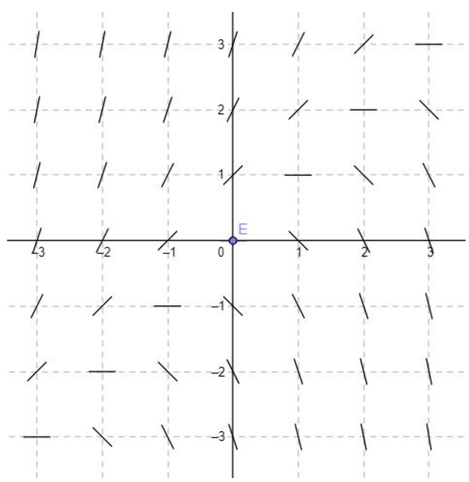
5. Draw a slope field for all integer coordinates $(x, y)$ with $-3 \leq x, y \leq 3$ for the following differential equations:

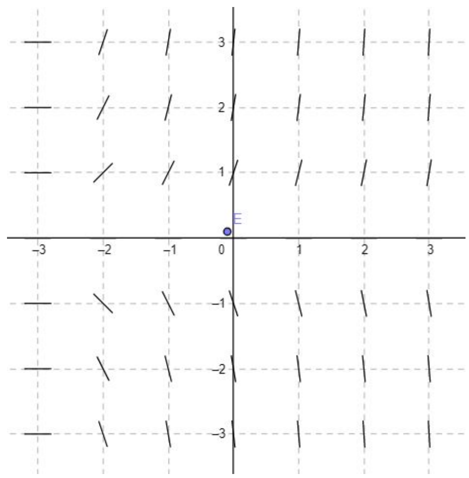
   (a) $y' = y - x$

   (b) $y' = 3y + xy$

   Solution: At each point with $x, y$ integers between $-3$ and 3, compute the slope according to the function $y' = f(x, y)$ and plot it on a graph as seen in the diagrams below.

(a)



(b) 

Answer Key

1. Compute the general solution to the differential equation $y' = x^2y - 2y$.

   $y = Ce^{x^3/3-2x}$.

2. Compute the general solution and particular solution to the differential equation $y' = x^3e^y$ with initial condition $y(0) = 1$.

   General solution: $y = \ln(-x^4/4 + C)$ Particular solution: $y = \ln(-x^4/4)$

3. Compute the general solution to the differential equation $y' = 2y(\sec^2 x - 1)$.

   $y = Ce^{2\tan x - 2x}$

4. Compute the general solution to the differential equation $yxy' = (x^2 - 1)$.

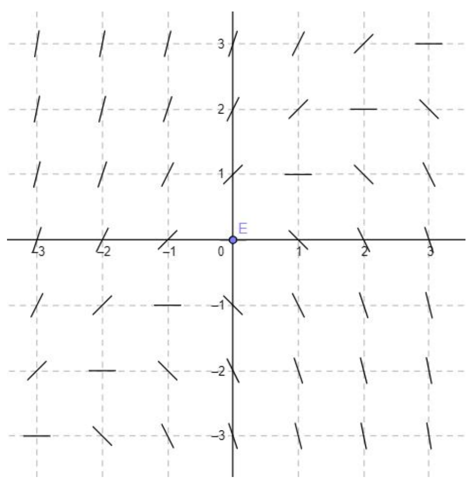
   $y^2 = x^2 - 2\ln|x| + C$

5. Draw a slope field for all integer coordinates $(x, y)$ with $-3 \le x, y \le 3$ for the following differential equations:

   (a) $y' = y - x$

   (b) $y' = 3y + xy$

(a)



(b)