MAT 132 HW 4-6

1. Problems

1. Compute the following indefinite integral:
$$\int \frac{17x - 53}{x^2 - 2x - 15}\, dx$$
2. Compute the following indefinite integral:
$$\int \frac{3x + 9}{(x+1)^2}\, dx$$
3. Compute the following indefinite integral:
$$\int \tan x \, dx$$
4. Compute the following indefinite integral:
$$\int \sin^4(x)\, dx$$
5. Find the average value of the function $f(x) = 4x^3 + 1$ on the interval $[-1, 3]$
6. Find the average value of the function $g(x) = 4e^{3x+1}$ on the interval $[0, 2]$.

2. Answer Key

1.
$$4\ln|x-5|+13\ln|x+3|+C$$
2.
$$3\ln|x+1|-\frac{6}{x+1}+C$$
3.
$$-\ln|\cos x|+C$$
5. 21
6. $\frac{2}{3}\left(e^5-e\right)$

3. Solutions

1. Factoring the denominator gives $x^2 - 2x - 15 = (x-5)(x+3)$. Performing partial fraction decomposition, we write
$$\frac{17x-53}{x^2-2x-15} = \frac{A}{x-5} + \frac{B}{x+3}$$
and we find that $A = 4$ and $B = 13$. Hence
$$\int \frac{17x-53}{x^2-2x-15}\,dx = \int \frac{4}{x-5} + \frac{13}{x+3}\,dx = 4\ln|x-5| + 13\ln|x+3| + C.$$
2. Performing partial fraction decomposition, we write
$$\frac{3x+9}{(x+1)^2} = \frac{A}{x+1} + \frac{Bx+C}{(x+1)^2}$$
and we find that $A = 3$, $B = 0$, and $C = 6$. Hence
$$\int \frac{3x+9}{(x+1)^2}\,dx = \int \frac{3}{x+1} + \frac{6}{(x+1)^2}\,dx = 3\ln|x+1| - \frac{6}{x+1} + C$$
3. We write $\tan x = \sin x/\cos x$ and perform u-substitution with $u = \cos x$. This gives
$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\cos x| + C.$$
4. We use the identities $\cos^2(x) = (1+\cos(2x))/2$ and $\sin^2(x) = (1-\cos(2x))/2$. Note that the second identity gives us
$$\sin^4(x) = \left[\sin^2(x)\right]^2 = \left[(1-\cos(2x))/2\right]^2.$$
It follows that
$$\begin{aligned}\int \sin^4(x)\,dx &= \int \left[\frac{1-\cos(2x)}{2}\right]^2\,dx \\ &= \frac{1}{4}\int 1 - 2\cos(2x) + \cos^2(2x)\,dx \\ &= \frac{1}{4}\int 1 - 2\cos(2x) + \frac{1}{2} + \frac{1}{2}\cos(4x)\,dx \\ &= \frac{1}{4}\left(x - \sin(2x) + \frac{1}{2}x + \frac{1}{8}\sin(4x)\right) + C \\ &= \frac{1}{4}\left(\frac{3x}{2} - \sin(2x) + \frac{1}{8}\sin(4x)\right) + C\end{aligned}$$
5.
$$\frac{1}{3-(-1)}\int_{-1}^{3} 4x^3 + 1\,dx = \frac{1}{4}\left(x^4 + x\right)\Big|_{-1}^{3} = \frac{84}{4} = 21$$
6.
$$\frac{1}{2-0}\int_{0}^{2} 4e^{3x+1}\,dx = \frac{2}{3}e^{2x+1}\Big|_{0}^{2} = \frac{2}{3}\left(e^5 - e\right).$$