MAT 132 HW 1-3

1. Problems

1. Compute the following indefinite integral:
$$\int \sin(7x+1)\,dx$$
2. Compute the following definite integral:
$$\int_5^{10} \frac{1}{\sqrt{x-1}}\,dx$$
3. Compute the following indefinite integral:
$$\int x\cos(5x+3)\,dx$$
4. Compute the following indefinite integral:
$$\int x^2 e^{-2x}\,dx$$
5. Compute the following indefinite integral:
$$\int t^{99}\ln t\,dt$$

2. Answer Key

1.
$$-\frac{1}{7}\cos(7x+1)+C$$
2. 2
3.
$$\frac{x\sin(5x+3)}{5}+\frac{\cos(5x+3)}{25}+C$$
4.
$$-\frac{1}{2}\left(x^2+x+\frac{1}{2}\right)e^{-2x}+C$$
5.
$$\frac{t^{100}\ln t}{100}-\frac{t^{100}}{10000}+C$$

3. Solutions

1. We let $u = 7x + 1$ so that $du = 7\ dx$. Then we have
$$\int \sin(7x+1)\ dx = \int \frac{1}{7}\sin u\ du = -\frac{1}{7}\cos u + C = -\frac{1}{7}\cos(7x+1) + C.$$
2. We let $u = x - 1$ so that $du = dx$. Our new limits of integration are $u = 10 - 1 = 9$ and $u = 5 - 1 = 4$. Then we have
$$\int_5^{10} \frac{1}{\sqrt{x-1}}\ dx = \int_4^9 \frac{1}{\sqrt{u}}\ du = 2\sqrt{u}\,\Big|_4^9 = 2.$$
3. Letting $u = x$ and $dv = \cos(5x+3)\ dx$ gives us $du = dx$ and $v = \frac{1}{5}\sin(5x+3)\ dx$. Hence we get
$$\int x\cos(5x+3)\ dx = \frac{x\sin(5x+3)}{5} - \frac{1}{5}\int \sin(5x+3)\ dx$$
This integral is handled using substitution. We let $s = 5x + 3$ so that $ds = 5\ dx$ and we have
$$\frac{1}{5}\int \sin(5x+3)\ dx = -\frac{\cos(5x+3)}{25} + C.$$
Hence the final answer is
$$\int x\cos(5x+3)\ dx = \frac{x\sin(5x+3)}{5} + \frac{\cos(5x+3)}{25} + C$$
4. We have to do integration by parts twice. First note that using the substitution $s = -2x$, the antiderivative of e^{-2x} is $-\frac{1}{2}e^{-2x}$. Letting $u = x^2$ and $dv = e^{-2x}\ dx$ gives us $du = 2x\ dx$ and $v = -\frac{1}{2}e^{-2x}$. This gives us
$$\int x^2e^{-2x}\ dx = -\frac{1}{2}x^2e^{-2x} + \int xe^{-2x}\ dx.$$
Again performing integration by parts with $u = x$ and $dv = e^{-2x}\ dx$ gives us
$$\int xe^{-2x}\ dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\ dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C.$$
Putting this all together, we get
$$\begin{aligned}\int x^2e^{-2x}\ dx &= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C\\ &= -\frac{1}{2}\left(x^2 + x + \frac{1}{2}\right)e^{-2x} + C.\end{aligned}$$
5. Let $u = \ln t$ and $dv = t^{99}\ dt$ so that $du = \frac{dt}{t}$ and $v = \frac{1}{100}t^{100}$. Then we get
$$\int t^{99}\ln t\ dt = \frac{t^{100}\ln t}{100} - \frac{1}{100}\int t^{99}\ dt = \frac{t^{100}\ln t}{100} - \frac{t^{100}}{10000} + C.$$