<u>Def</u>. A <u>sequence</u> is an infinite ordered list of numbers
having the 1st el-t but no last el-t.

<u>Notation</u> $\{ a_1, a_2, a_3, \ldots, a_n, \ldots \} = \{ a_n \}_{n=①}^{\infty}$

↑ ↑   ↑   general
1st term 2nd term n^{th} term   term

<u>Ex</u>. $\{ 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \} = \{ \frac{1}{2^n} \}_{n=0}^{\infty}$ geom. sequence

     ‖ ‖
     a_0 a_1 ↑
                         general term

Compare a sequence and a f-n

$$\mathbb{Z}^+ \longrightarrow \mathbb{R}$$
$$n \longmapsto \underset{\uparrow}{a_{\textcircled{n}}}$$
variable

$$\left. \begin{array}{c} \end{array} \right\} \quad f: \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto f(x)$$
            ↑
          variable

A sequence is a f-n whose
domain is $\mathbb{Z}^+$.

Compare seq. $\{ \frac{1}{2^n} \}_{n=0}^{\infty}$ and f-n $f(x) = \frac{1}{2^x}$

$$f(n) = \frac{1}{2^n} = a_n$$

$y = \frac{1}{2^x}$

A f-n $f(x)$ where $f(n) = a_n$ for all $n \in \mathbb{Z}^+$ is
called an <u>extension</u> of $\{ a_n \}_{n=1}^{\infty}$

<u>How to describe a sequence?</u>

1) By a formula for a <u>general term</u>

<u>Ex</u>. $\{ (-1)^n n^2 \}_{n=1}^{\infty} = \{ -1, 4, -9, 16, \ldots \}$

      <u>gen. term</u> $n=1$ $n=2$ $n=3$

2) By a recursive formula

Ex. 1
$$\begin{cases} a_1 = 1 \\ a_{n+1} = a_n + 3 \end{cases}, \quad n = 1, 2, 3 ..$$

$\{ 1, 4, 7, 10, ... \}$ arithmetic sequence

$a_2 = a_1 + 3$
$a_3 = a_2 + 3$

Ex. 2
$$\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_{n+1} = a_n + a_{n-1} \end{cases}, \quad n = 2, 3, ..$$

$\{ 1, 1, 2, 3, 5, 8, 13, 21, ... \}$ Fibonacci sequence

$a_3 = a_2 + a_1$
$a_4 = a_3 + a_2$