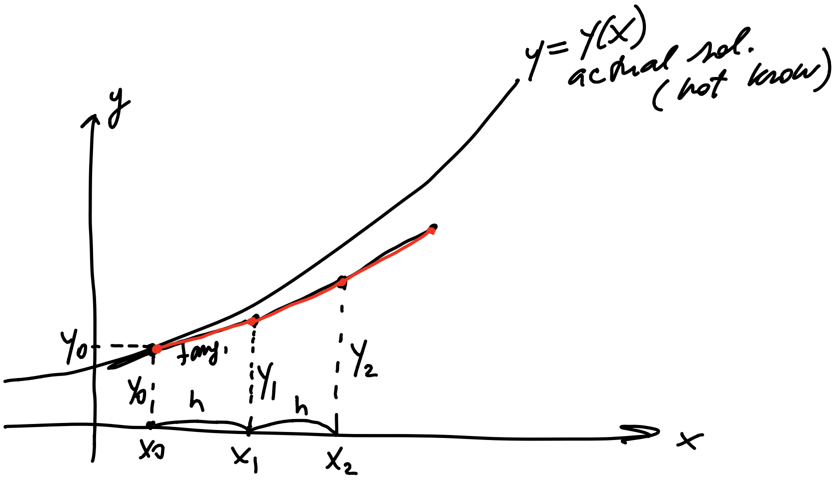
Episode 20: Euler's method

a numerical sol. of IVP

$$\text{IVP} \begin{cases} \boxed{y' = f(x,y)} \\ y(x_0) = y_0 \end{cases}$$



Given: f-n f(x,y)

numbers x_0, y_0

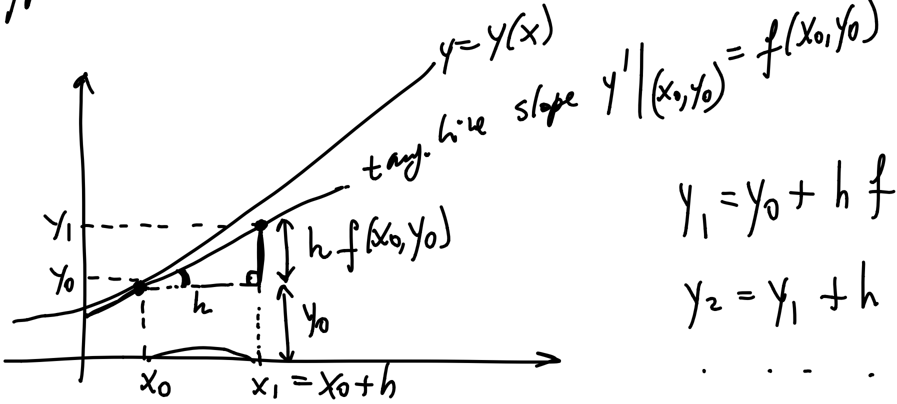
step h

Find: broken line

$(x_0,y_0), (x_1,y_1), (x_2,y_2), \ldots, (x_n,y_n)$

approximate the sol. curve

Sol.



$$y_1 = y_0 + h\,f(x_0,y_0)$$

$$y_2 = y_1 + h\,f(x_1,y_1)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$y_n = y_{n-1} + h\,f(x_{n-1}, y_{n-1})$$

$$\boxed{\begin{array}{c} y_{k+1} = y_k + h\,f(x_k, y_k) \\ k = 0, 1, \ldots, n-1 \end{array}}$$

Ex. Use Euler's method with step of size $\boxed{0.1}$ to estimate $y(0.3)$ where $y(x)$ is the sol. of IVP

$$\begin{cases} y' = \boxed{x+y}\ f(x,y) \\ y(\underset{\substack{\uparrow \\ x_0}}{0}) = \underset{\substack{\uparrow \\ y_0}}{1} \end{cases}$$

Sol.

Given: $f(x,y) = \underline{x+y}$

$x_0 = 0$

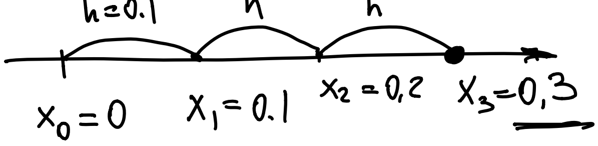
$y_0 = 1$

$h = 0.1$

Find $y(0,3)$

$\|$

$y(x_3)$



$$y_{k+1}=y_k+h\,f(x_k,y_k)$$

$k=0$: $y_1=y_0+h\,f(x_0,y_0)=$

$1+0.1\cdot1=1.1$

$f(x_0,y_0)=f(0,1)=0+1=1$

$k=1$: $y_2=y_1+h\,f(x_1,y_1)=$

$1,1+0,1\cdot1,2=1.1+0.12=1,22$

$f(x_1,y_1)=f(0.1,1.1)=0.1+1,1=$ $1,2$

$k=2$: $y_3=y_2+h\,f(x_2,y_2)=$

$1,22+0,1\cdot1,42=$

$1.22+0.142=\boxed{1.362}$

$f(x_2,y_2)=f(0.2,1.22)=$ $0,2+1,22=1.42$

Answer: $y(0,3)\approx1,362$

Remark This IVP has exact sol.

$y(x)=-(x+1)+2e^x$

$y(0,3)\approx1.3997\ldots$