Episode 18: Direction fields and solution curves

Geometrically, to solve a DE = find its

solution curves / integral lines / field lines

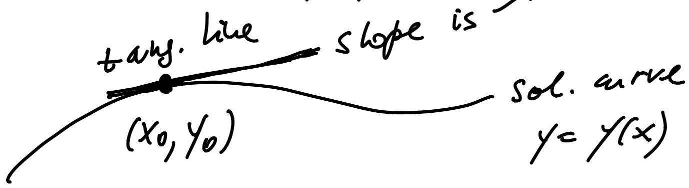
How to see sol. curve without solving DE?

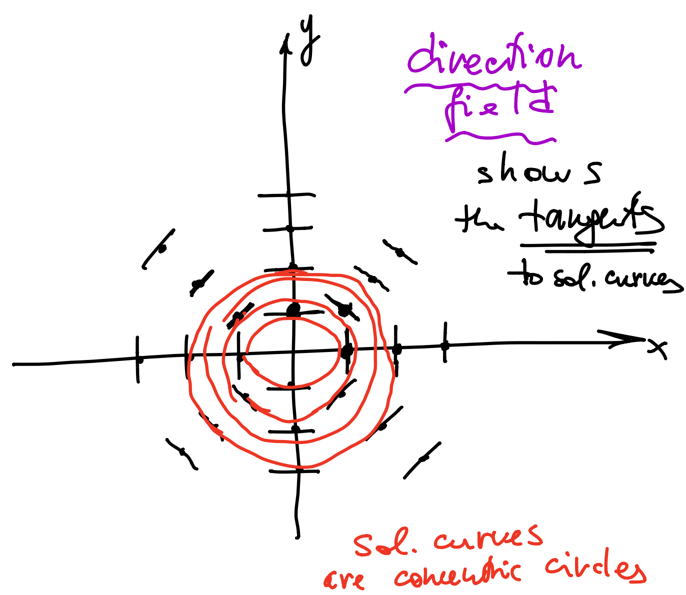
Ex. DE $x + yy' = 0$

Solve for y' : $\underbrace{y'}_{\text{slope of sol. curve } y=y(x)} = -\frac{x}{y}$ (y' depends on x, y)

For any pt (x_0, y_0) on a sol. curve $y = y(x)$

$$y'\Big|_{\substack{x=x_0\\y=y_0}} = -\frac{x_0}{y_0}$$





Plug in some value of (x_0, y_0):

$$y'\Big|_{\substack{x=0\\y=1}} = -\frac{x}{y}\Big|_{\substack{x=0\\y=1}} = -\frac{0}{1} = 0$$

(for all pt $(0, y)$, $y' = 0$)

$$y'\Big|_{\substack{x=1\\y=0}} = \left[-\frac{1}{0}\right]$$ inf. slope vertical target

(for all pt $(x, 0)$, the tang. is vert.)

$$y'\Big|_{\substack{x=1\\y=1}} = -\frac{1}{1} = -1$$

(for all pts (x, x), $y' = -1$)

$$y'\Big|_{\substack{x=-1\\y=1}} = -\frac{-1}{1} = 1$$

Solve this DE exactly:

$$y' = -\frac{x}{y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

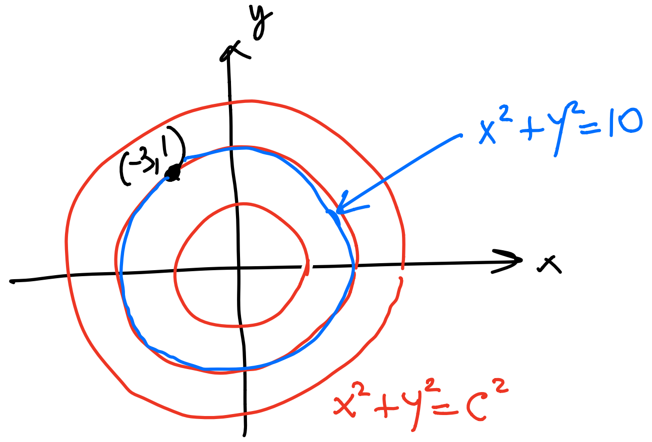
$$y\,dy = -x\,dx$$

$$\int y\,dy = -\int x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + C_1$$

general sol. of DE $\boxed{x^2+y^2 = C^2}$ $\quad C^2 = 2C_1$

implicit sol. of the DE



Extra $\rightarrow$ IVP $\begin{cases} y' = -\frac{x}{y} \\ y(\underset{x}{\underset{\uparrow}{-1}}) = \underset{y}{\underset{\uparrow}{3}} \end{cases}$

1) Solve the DE:

$$x^2+y^2 = C^2$$

2) Find C

$$(-1)^2 + 3^2 = C^2 \Rightarrow C^2 = 10$$

Sol. for the IVP is

$$\boxed{x^2+y^2 = 10}$$