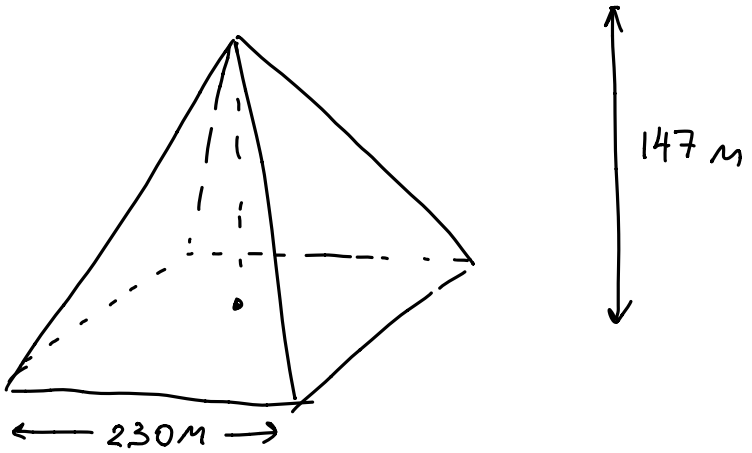
Episode 16.

How much work was needed to erect the Great Pyramid of Giza?



Find the work against the gravity to construct this pyramid

Given: hight: $h = 147\,m$

base: $a = 230\,m$

density of lime stone: $\rho = 2.5 \cdot 10^3 \frac{kg}{m^3}$

Find: Work against the force of gravity

From Physics: $W = F \cdot S$

work force disp.

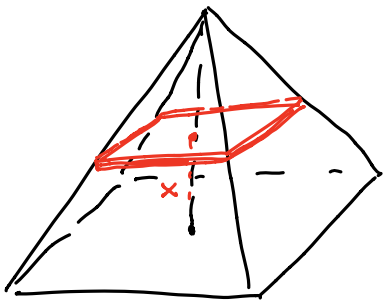
weight = force of gravity

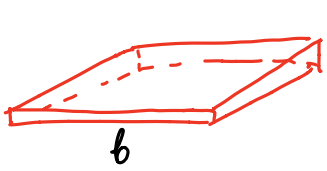
$F = mg$

$m = \rho V$

$V =$

Consider all stones at the same hight



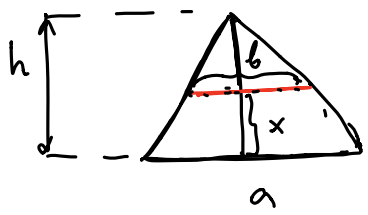


horizontal slab at light x

Work to lift this slab to light x=?

dW = ?

Vertical cross-sec. of the pyramid



From similar Δs:

$$\frac{b}{a} = \frac{h-x}{h} \Rightarrow b = \frac{a}{h}(h-x)$$

The volume of the slab is $dV = \underbrace{b^2}_{\text{area}}\underbrace{dx}_{\text{thickness}} = \frac{a^2}{h^2}(h-x)^2\,dx$

The mass of the slab is $dm = \rho\, dV = \frac{\rho a^2}{h^2}(h-x)^2\,dx$

The weight of the slab is $dF = dm \cdot g = \frac{\rho g a^2}{h^2}(h-x)^2\,dx$

The work against the gravity to lift the slab at light x is

$$dW = \underbrace{dF}_{\text{weight}} \cdot \underbrace{x}_{\text{light}} = \frac{\rho g a^2}{h^2}(h-x)^2 \cdot x\,dx$$

Total work to erect the pyramid is

$$W = \int_{x=0}^{x=h} dW = \int_0^h \frac{\rho g a^2}{h^2}(h-x)^2 x\,dx =$$

$$= \frac{\rho g a^2}{h^2}\int_0^h (x^3 - 2hx^2 + h^2x)\,dx =$$

$$= \frac{\rho g a^2}{h^2}\left(\frac{h^4}{4} - \frac{2h^4}{3} + \frac{h^4}{2}\right) = \frac{\rho g a^2 h^2}{12} = \text{plug in numbers}$$

$$= 2.3 \cdot 10^{12}\ (J)$$

$1\ J$ = watt · sec $\qquad 2.3 \cdot 10^{12}\ J = 6.4 \cdot 10^5$ kwh