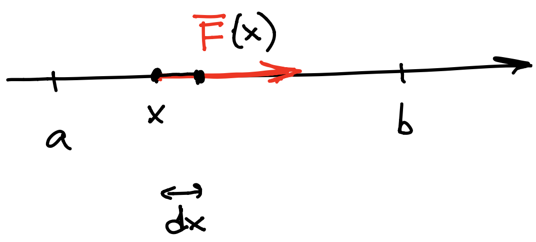
Episode 15: Mechanical work

work = force × displacement

Assume that an object moves along a straight line and the force is a f-n of the position of the object.



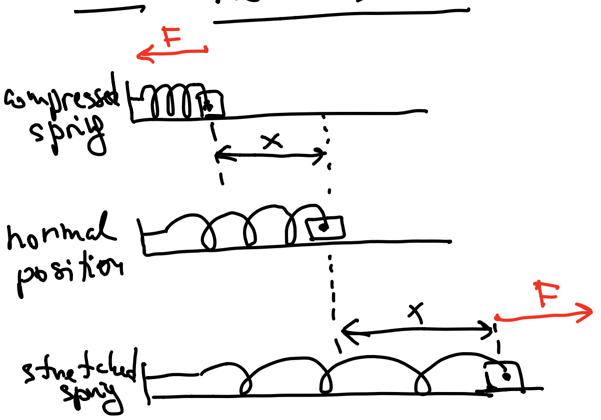
On small displ. dx, $F(x) = \text{const}$ and

$$\underbrace{dW}_{\text{work el-t}} = \underbrace{F(x)}_{\text{force}} \cdot \underbrace{dx}_{\text{displ.}}$$

Total work:

$$W = \int dW = \boxed{\int_{x=a}^{x=b} F(x)\,dx}$$

Ex. Hooke's law

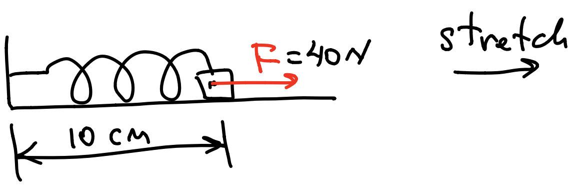


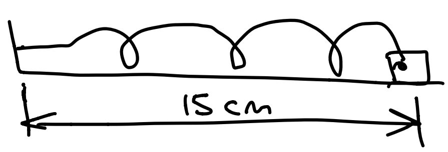
$$\boxed{\underbrace{F(x)}_{\text{force}} = \overset{\uparrow}{k}\,\underbrace{x}_{\text{displacement}}}$$

k – spring const

Problem. A force of 40 N is required to stretch a spring from its natural length of 10 cm to the length of 15 cm. How much work is needed to stretch a spring from 15 cm to 18 cm?

Sol.





By Hooke's law $F(x) = k\underbrace{x}_{\text{displ.}}$

displacement $x = 15\text{ cm} - 10\text{ cm} = 5\text{ cm} = 0.05\text{ m}$ (SI)

$F(0.05) = k \cdot 0.05 \Rightarrow k = \frac{40}{0.05} = \frac{4000}{5} = \underline{800}$

($F(0.05) = 40$)

So $F(x) = kx = \underline{800x}$

$$\text{work} = \int_{0.05}^{0.08} F(x)\,dx = \int_{0.05}^{0.08} 800x\,dx = 400x^2 \Big|_{0.05}^{0.08} = 1.56\,(J)$$

(0.05, 0.08: displacements from nat. pos.)

units: $[W] = [F \cdot x] = N \cdot m = J$