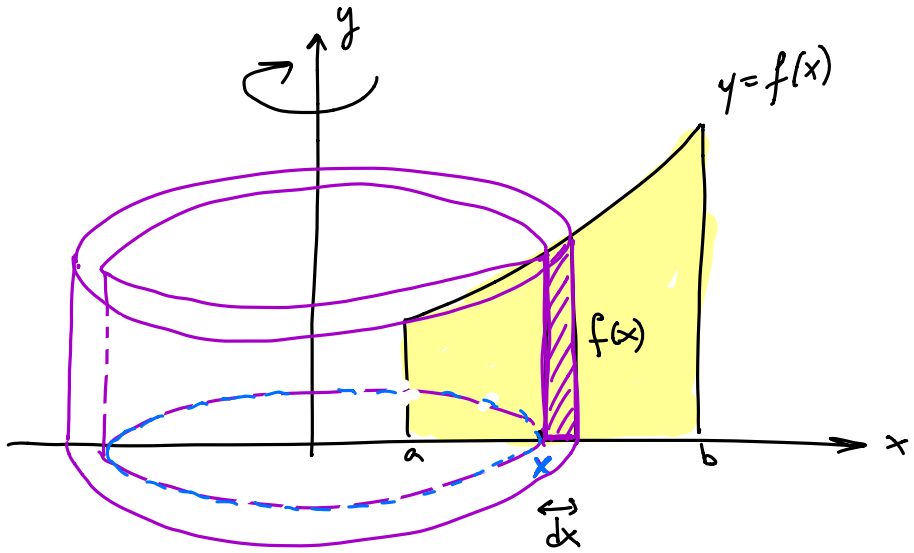
Episode 14: Volumes by cylindrical shells

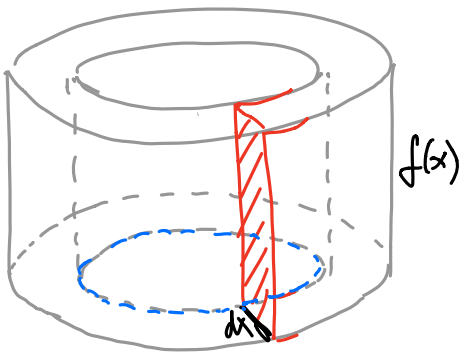


Vol of solid of revolution V = ?

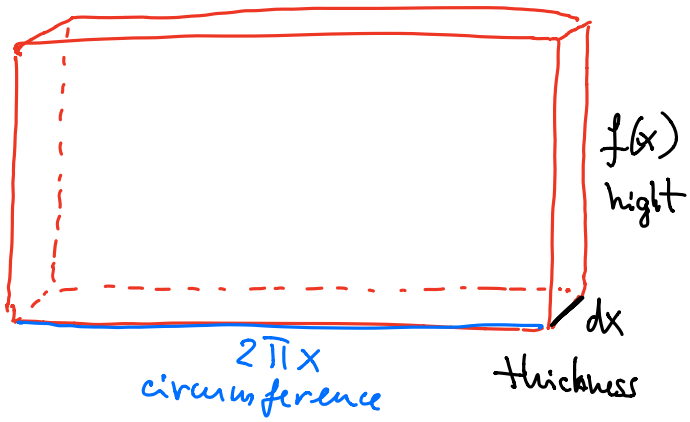
$V = \int dV$

dV = vol of cylindrical shell

space between two cylinders



cut and open



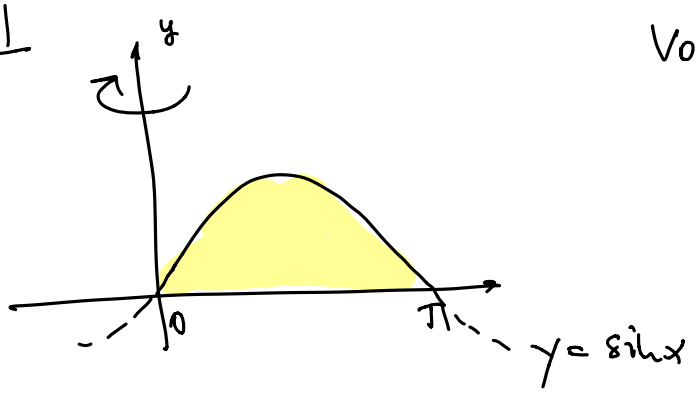
$$dV = \underbrace{2\pi x}_{\text{circumference}} \cdot \underbrace{f(x)}_{\text{hight}} \cdot \underbrace{dx}_{\text{thickness}}$$

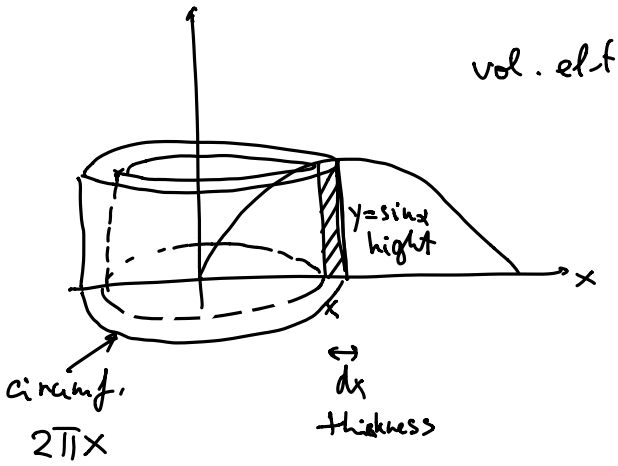
$$V = \int dV = 2\pi \int_{x=a}^{x=b} x\, f(x)\, dx$$

Vol by cylindrical shells

Ex. 1

Vol = ?



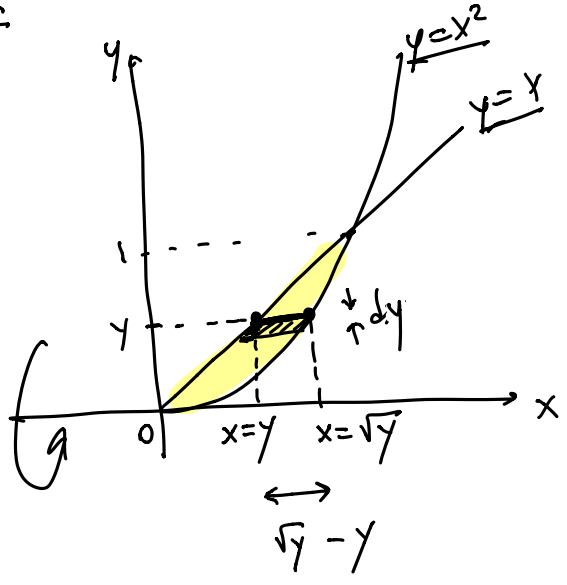


vol. el-t $dV = \underbrace{2\pi x}_{\text{circumf.}} \cdot \underbrace{\sin x}_{\text{height}} \cdot \underbrace{dx}_{\text{thickness}}$

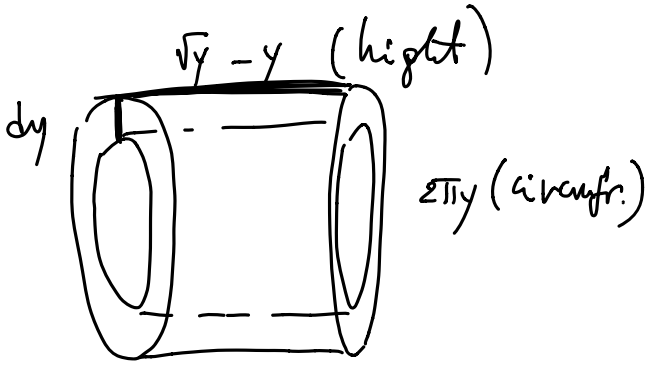
$$V = \int dV = 2\pi \int_0^{\pi} x \underbrace{\sin x}_{-\cos x} dx = 2\pi \left(-x\cos x \Big|_0^{\pi} + \int_0^{\pi} \cos x \, dx \right) =$$

$$= 2\pi \left(-\pi(-1) + \sin x \Big|_0^{\pi} \right) = \boxed{2\pi^2} \quad \text{cubic units}$$

Ex. 2



Vol = ?

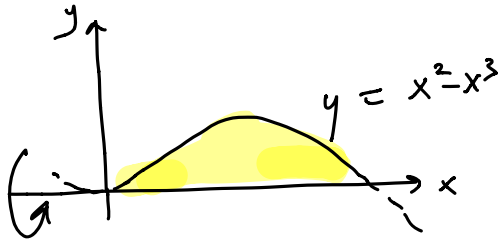


$dV = \underbrace{2\pi y}_{\text{circumf.}} \cdot \underbrace{(\sqrt{y} - y)}_{\text{height}} \underbrace{dy}_{\text{thick.}}$

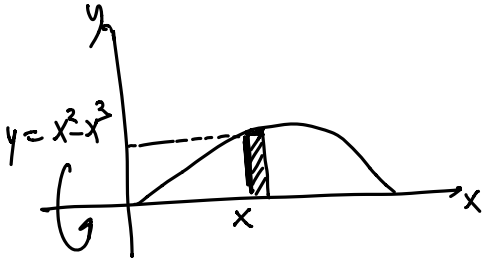
$$V = \int dV = 2\pi \int_{y=0}^{y=1} y(\sqrt{y} - y)\, dy = 2\pi \int \left(y^{3/2} - y^2 \right) dy =$$

$= 2\pi \left(\frac{2}{5} y^{\frac{5}{2}} - \frac{1}{3} y^3 \right)_0 = 2\pi \left(\frac{2}{5} - \frac{1}{3} \right) = \boxed{\frac{2\pi}{15}}$ cubic units

By slicing or by cyl. shells ?

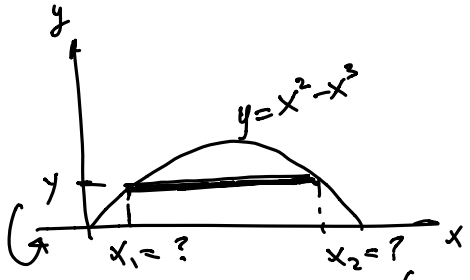


by slicing



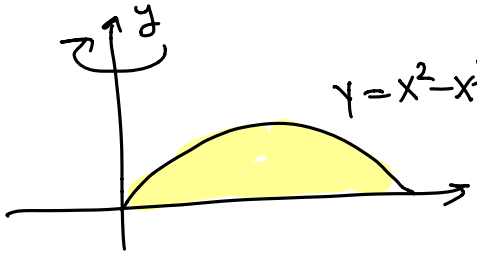
$dV = A(x)\,dx = \pi \left(x^2 - x^3 \right)^2 dx$

by cyl. shells

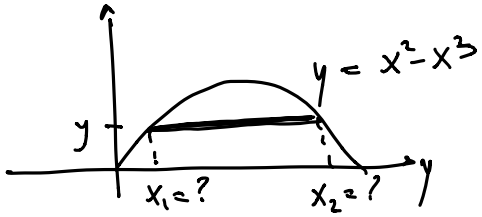


$dV = \underbrace{2\pi y}_{\text{circumf.}} \cdot \underbrace{(x_2 - x_1)}_{\text{height}} \cdot \underbrace{dy}_{\text{thickness}}$

?



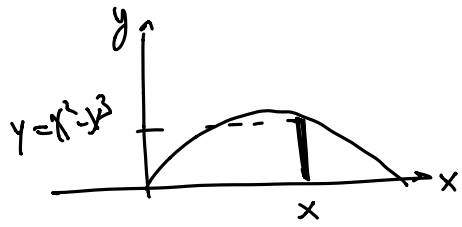
by slicing



$dV = A(x)\,dx = \pi \left(x_2^2 - x_1^2 \right) dx$

?

by cyl. shells



$dV = \underbrace{2\pi x}_{\text{circumf.}} \underbrace{\left(x^2 - x^3 \right)}_{\text{height}} \cdot \underbrace{dx}_{\text{thickness}}$