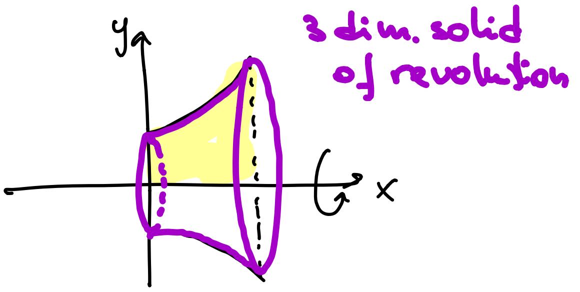
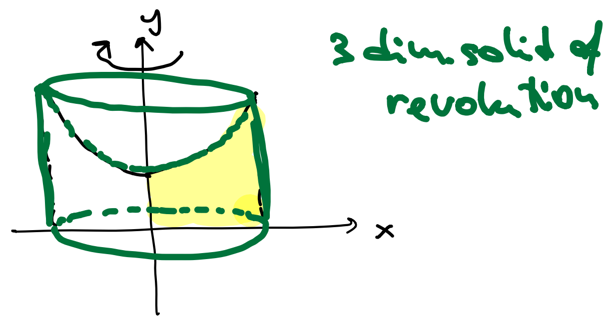
Episode 13 : Volumes by slicing

A solid of revolution is a solid obtained when a region is rotated around an axis



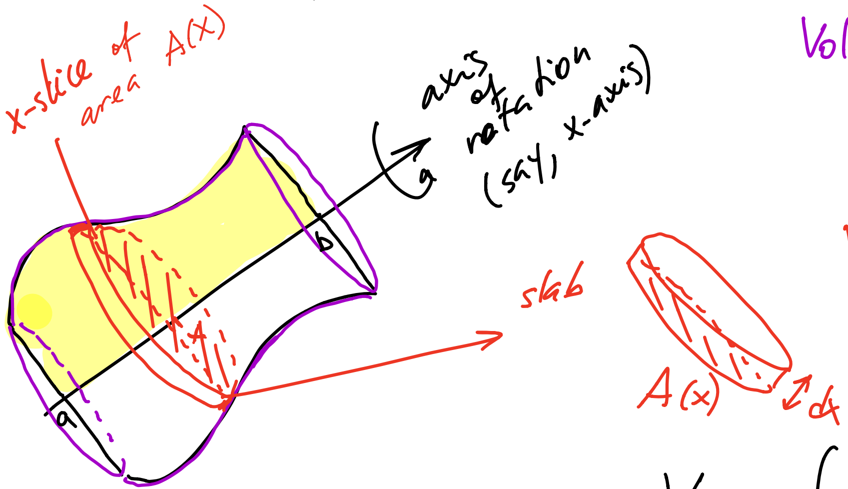


Volume

by slicing — by cylindrical shells

Volume by slicing

Vol = ?



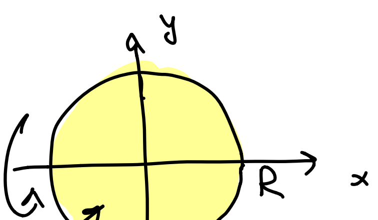
Vol is $dV = A(x)\,dx$

Total vol is $V = \underbrace{\int dV}_{\text{vol element}} = \int_{x=a}^{x=b} A(x)\,dx$

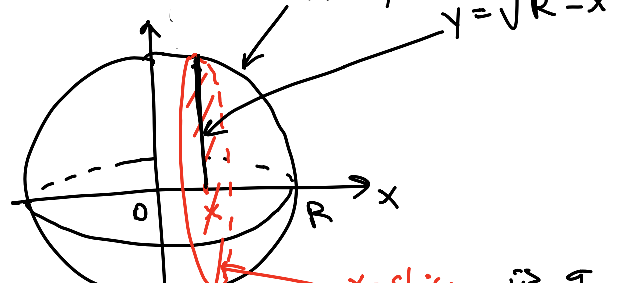
$$\boxed{V = \int dV = \int_a^b A(x)\,dx}$$

vol by slicing

Ex.) Vol of a ball of radius $R = ?$



rotate around the x-axis



disk
$x^2+y^2 \le R^2$

disk of rad. $\sqrt{R^2-x^2}$

Its area is $\pi\left(\sqrt{R^2-x^2}\right)^2 = \pi(R^2-x^2)$

The vol el-t is

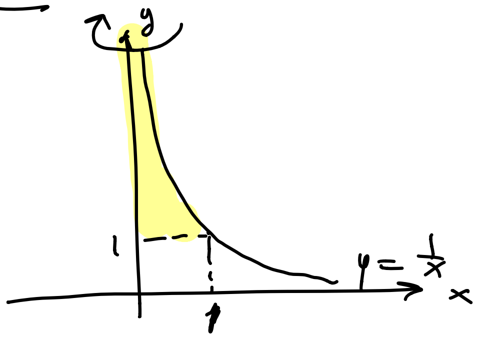
$$dV = A(x)\,dx$$

$$\frac{1}{2}V = \int dV = \int_{x=0}^{x=R} A(x)\,dx = \int_0^R \pi(R^2-x^2)\,dx =$$

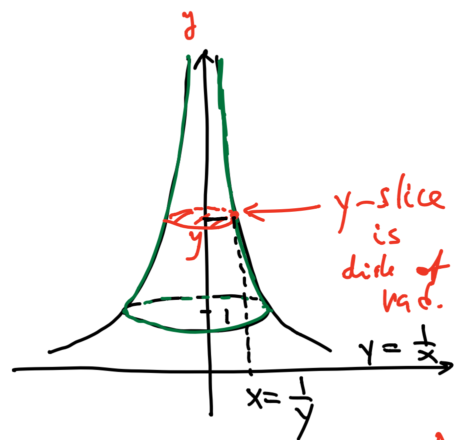
$$\pi\left(R^2x - \frac{1}{3}x^3\right)\Big|_0^R = \pi\left(R^3 - \frac{1}{3}R^3\right) = \frac{2\pi R^3}{3}$$

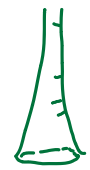
$$\boxed{V = \frac{4}{3}\pi R^3}$$ vol of a ball of rad. R

Ex. 2



rotate around y-axis

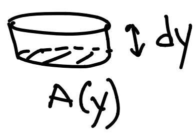




Vol = ?

Its area is $A(y) = \pi\left(\frac{1}{y}\right)^2$

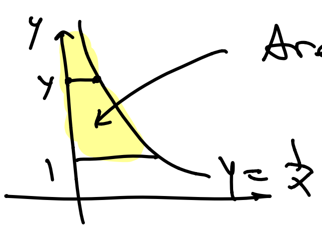
Vol el-t is $dV = A(y)\,dy$ (vol of y-slab)



$$\text{Vol} = \int dV = \int_{y=1}^{\infty} A(y)\,dy =$$

$$= \int_1^\infty \pi\left(\frac{1}{y}\right)^2 dy = -\frac{\pi}{y}\Big|_1^\infty = -\pi(0-1) = \pi$$ finite volume!

Remark:



$$\text{Area} = \int_1^\infty \frac{1}{y}\,dy = \ln y\Big|_1^\infty = \infty$$ div. to ∞