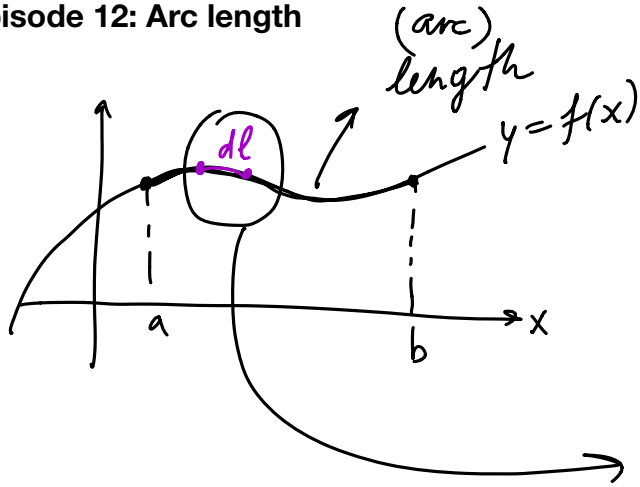
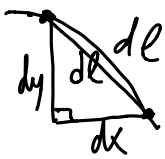


arc length el-t

$$dl \approx \Delta l = \sqrt{(dx)^2 + (dy)^2}$$



Total length

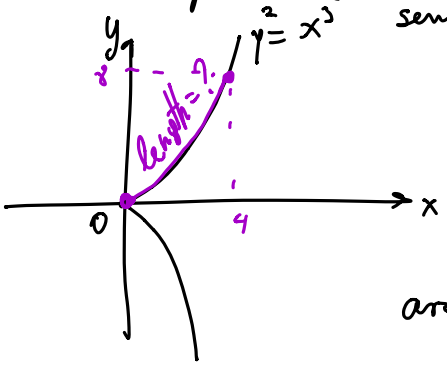
$$l = \int dl = \int_{x=a}^{x=b} \sqrt{(dx)^2 + (dy)^2} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx =$$

$$= \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\, dx$$

arc length

$$\boxed{l = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\, dx}$$

Ex. Find the arc length of the curve $y^2 = x^3$ from $(0,0)$ and $(4,8)$



semicubical parabola

$$y^2 = x^3 \Rightarrow y = x^{\frac{3}{2}} \quad (y \geq 0)$$

$$y' = \frac{3}{2} x^{\frac{1}{2}}$$

$$\text{arc length} = \int_{x=0}^{x=4} \sqrt{1 + (y')^2}\, dx = \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx =$$

$$\left[\begin{array}{l} u = 1 + \frac{9}{4}x \\ du = \frac{9}{4}\, dx \\ x = 0 \Rightarrow u = 1 \\ x = 4 \Rightarrow u = 10 \end{array}\right] = \int_1^{10} \sqrt{u}\, \frac{4}{9}\, du = \frac{4}{9} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^{10} = \frac{8}{27}\left[10^{\frac{3}{2}} - 1\right]$$