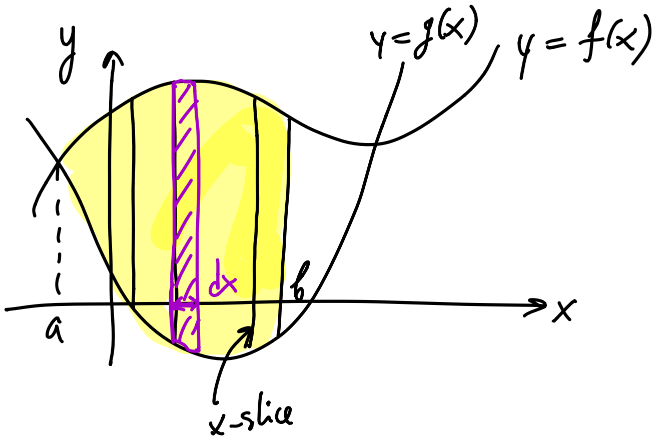
Episode 9: Area between curves

Applications of integration

Geometry	Physics	Differential Equations
area volume arc length	work	

Area between curves on Cartesian plane

By x-slices



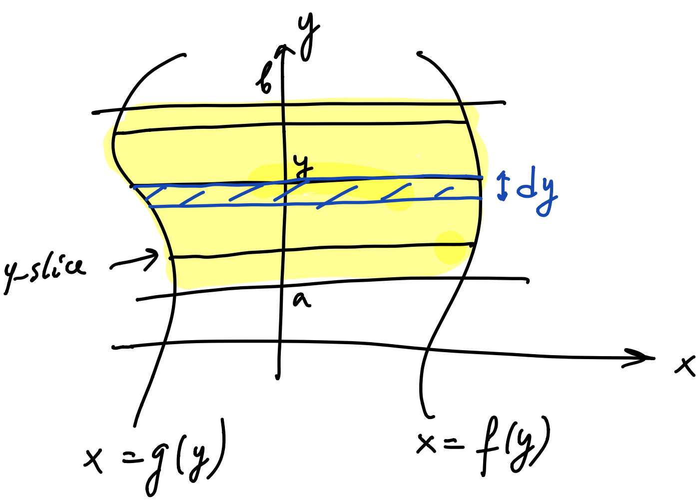
area element

$$dA = \underbrace{\big(f(x)-g(x)\big)}_{\text{hight}}\underbrace{dx}_{\text{base}}$$

total area

$$A = \int_a^b dA = \int_a^b \underbrace{\big(\underset{\text{upper}}{f(x)} - \underset{\text{lower}}{g(x)}\big)}_{\text{length of x-slice}} dx$$

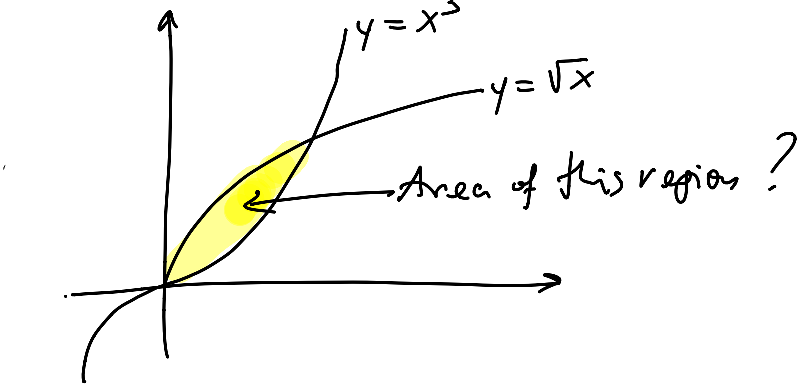
By y-slices



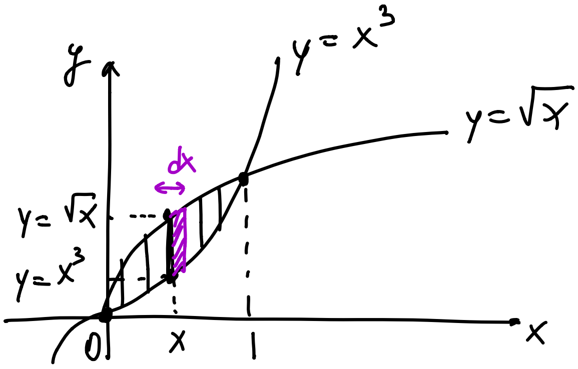
$$\underbrace{dA}_{\text{area el-t}} = \underbrace{\big(f(y)-g(y)\big)}_{\substack{\text{base} \\ \text{(length of y-slice)}}}\underbrace{dy}_{\text{hight}}$$

$$A = \int_{y=a}^{y=b} dA = \int_{y=a}^{y=b} \big(\underset{\text{right}}{f(y)} - \underset{\text{left}}{g(y)}\big)\, dy$$

Ex. Find the area of a bounded region enclosed by curves $y = x^3$ and $y = \sqrt{x}$.



Alt. 1 (by x-slices)



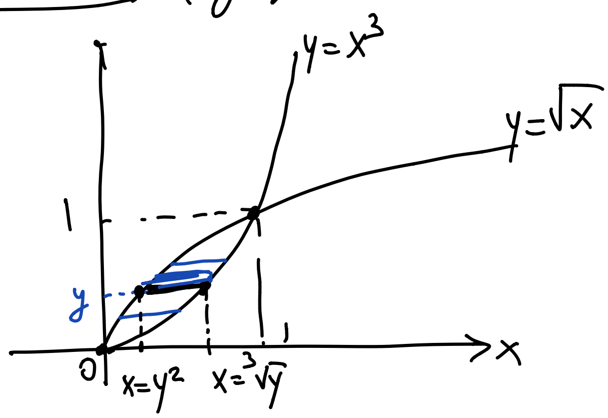
$$dA = (\sqrt{x} - x^3)\,dx$$

$$A = \int_0^1 dA = \int_0^1 (\sqrt{x} - x^3)\,dx =$$

$$\left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{4}x^4\right]_0^1 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

sq. units

Alt. 2 (by y-slices)



$$dA = (\sqrt[3]{y} - y^2)\,dy$$

$$A = \int_{y=0}^{y=1} (\sqrt[3]{y} - y^2)\,dy =$$

$$= \left[\frac{3}{4}y^{\frac{4}{3}} - \frac{1}{3}y^3\right]_0^1 = \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$$

sq. units