Episode 6: Average value of a function

discrete variable: the ave value of

$$\underbrace{y_1, \dots, y_n}_{n \text{ numbers}} \quad \text{is} \quad y_{ave} = \frac{y_1+y_2+\dots+y_n}{n}$$

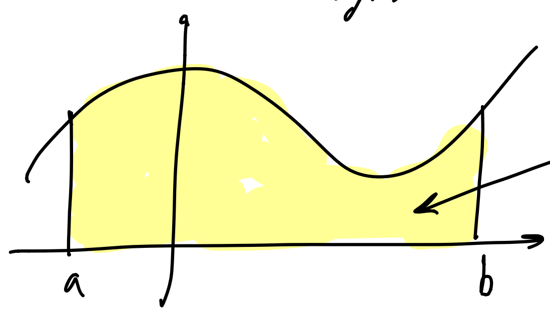
continuous variable: $y=f(x)$ on $[a,b]$

Ave value of $y=f(x)$ on $[a,b]$ is

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx} \qquad \text{(number)}$$

Geom. meaning: If $f \geq 0$

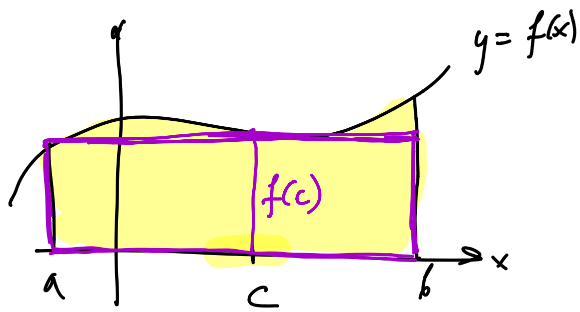
$$f_{ave} = \frac{\text{Area}}{b-a} = \text{area per unit length} \qquad \int_a^b f(x)\,dx = \text{Area}$$



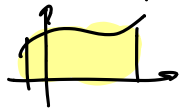
Mean Value Theorem for integrals

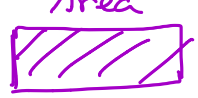
If f is a cont. f-n on $[a,b]$ then there exists $c \in [a,b]$ s.t.

$$f(c) = \underbrace{\frac{1}{b-a}\int_a^b f(x)\,dx}_{f_{ave}}$$



$$\Leftrightarrow \quad \underbrace{\int_a^b f(x)\,dx}_{\text{Area}} = \underbrace{f(c)\cdot(b-a)}_{\text{Area}}$$





Ex. $f(x) = x^2$ Find f_{ave} on $[-1, 1]$?

$$f_{ave} = \frac{1}{1-(-1)} \int_{-1}^{1} x^2 \, dx = \frac{1}{2} \cdot 2 \int_{0}^{1} x^2 \, dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3}$$

$f(x) = x^2$ is even