

Episode 3: Integration by parts

$$u = u(x)$$

$$v = v(x)$$

product rule $\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$

↓ integrate

$$uv = \int \frac{du}{dx} v dx + \int u \frac{dv}{dx} dx$$

$$uv = \int v du + \int u dv$$

$$\boxed{\int u dv = uv - \int v du} \quad \text{int. by parts}$$

for def. int.

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

A diagram illustrating the integration by parts formula. It shows the integral $\int u dv$ with a purple arrow pointing from u to du and a blue arrow pointing from dv to v . The result is $uv - \int v du$, with uv underlined in purple.

$$\int u dv = \underline{uv} - \int v du$$

Ex. 1 $\int x e^x dx$

3 ways to write:

1) $\int \underbrace{x}_u \underbrace{e^x dx}_{dv} \left[\begin{array}{ll} u = x & dv = e^x dx \\ du = dx & v = e^x \end{array} \right] =$

$$\frac{x}{u} \frac{e^x}{v} - \int \frac{e^x}{v} \frac{dx}{du} = \boxed{x e^x - e^x + C}$$

$$2) \int x e^x dx = \int \underbrace{x}_u d(\underbrace{e^x}_v) = \underbrace{x e^x}_{uv} - \int \underbrace{e^x}_v \underbrace{1}_{\frac{du}{dx}} dx = \underline{x e^x - e^x + C}$$

$$3) \int \underbrace{x}_u \underbrace{e^x}_v dx = \underbrace{x e^x}_{uv} - \int \underbrace{e^x}_v \underbrace{1}_{\frac{du}{dx}} dx = \underline{x e^x - e^x + C}$$

Ex. 2 (int. by parts twice)

$$\int \underbrace{x^2}_u \underbrace{\cos x}_v dx = x^2 \sin x - \int \sin x \cdot 2x dx = x^2 \sin x - 2 \int \underbrace{x}_u \underbrace{\sin x}_v dx =$$

$$= x^2 \sin x - 2 \left[x (-\cos x) - \int (-\cos x) dx \right] =$$

$$\underline{x^2 \sin x + 2x \cos x - 2 \sin x + C}$$

Ex. 3 (a trick)

$$\int \ln x dx = \int \underbrace{\ln x}_u \cdot \underbrace{1}_v dx = x \ln x - \int \underbrace{x}_u \cdot \underbrace{\frac{1}{x}}_v dx =$$

$$x \ln x - \int dx = \underline{x \ln x - x + C}$$

Ex. 4 (int. by part + subs)

$$\int_0^1 \arctan x dx = \int_0^1 \underbrace{\arctan x}_u \cdot \underbrace{1}_v dx = x \arctan x \Big|_0^1 - \int_0^1 \underbrace{x}_u \underbrace{\frac{1}{1+x^2}}_v dx =$$

$$= 1 \cdot \arctan 1 - \frac{1}{2} \ln 2 = \left[\frac{\pi}{4} - \frac{1}{2} \ln 2 \right]$$

$$\int_{x=0}^{x=1} \frac{x}{1+x^2} dx = \left[\begin{array}{l} u = 1+x^2 \\ du = 2x dx \Rightarrow x dx = du/2 \\ x=0 \Rightarrow u = 1+0^2 = 1 \\ x=1 \Rightarrow u = 1+1^2 = 2 \end{array} \right] = \int_{u=1}^{u=2} \frac{du/2}{u} =$$

$$= \frac{1}{2} \ln |u| \Big|_1^2 = \frac{1}{2} \ln 2$$