MAT131 Homework for Lectures 25-26

July 16, 2021

1 Problems

1. Compute $\int x^{2020} - 3\sin(5x)\,dx$. Check your answer with differentiation.
2. Compute $\int e^{\pi x} - \sec^2(x)\,dx$. Check your answer with differentiation.
3. Find the antiderivative of the antiderivative of the antiderivative of $f(x) = x^7 - x^5 + \frac{1}{x^3}$.
4. Find the derivative of $y = \tan^{-1}(x)$ using implicit differentiation.
5. Find the antiderivative of $g(u) = (u^2 + 1)^{-1}$.
6. Compute $\int \sin^2\theta\,d\theta$ and check your answer by differentiation.

2 Answer Key

1. $\frac{x^{2021}}{2021} + \frac{3}{5}\cos(5x) + C$
2. $\frac{e^{\pi x}}{\pi} - \tan(x) + C$
3. $F(x) = \frac{x^{10}}{720} - \frac{x^8}{336} + \frac{1}{2}\ln(x) + C$
4. $\frac{dy}{dx} = \frac{1}{1+x^2}$
5. $G(u) = \tan^{-1}(u) + C$
6. $\frac{1}{2}(\theta - \frac{1}{2}\sin(2\theta)) + C$

3 Solution

1. Just use the power rule for the first term and think about the derivatives of $\sin(x)$ and $\cos(x)$ for the second term. Those derivatives (up to constants) have a periodic behavior.

2. One just needs to remember that $\frac{d}{dx}\tan(x) = \sec^2 x$.

3. Apply power rule 3 times. The last time, recall that $\frac{d}{dx}\ln(x) = 1/x$.

4. Rewrite the equation as $\tan y = x$. Then implicit differentiation gives $\sec^2(y)\frac{dy}{dx} = 1$ so $\frac{dy}{dx} = \cos^2(y)$. We may reinterpret $\tan(y) = x$ as saying that the tangent of the angle y of a right triangle is $x/1$; i.e. the opposite over the adjacent. Hence, the hypotenuse has length $\sqrt{1+x^2}$. The cosine of the angle is adjacent over hypotenuse; squaring this, we get $\cos^2(y) = \frac{1}{1+x^2}$.

5. Question 3 basically gives the answer: $G(u) = \tan^{-1}(u) + C$.

6. Use the trig identity $\cos(2\theta) = 1 - 2\sin^2\theta$. So we're integrating: $\int \frac{1}{2}(1 - \cos(2\theta))\, d\theta$. So then, this equals $\frac{1}{2}(\theta - \frac{1}{2}\sin(2\theta)) + C$.