MAT 127 HW 26-28

1. Problems

1. Determine if the following series converges.
$$\sum_{n=1}^{\infty} \left(\frac{\ln(n)}{n}\right)^4.$$
2. Determine if the following series converges absolutely, conditionally, or not at all.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+2}.$$
3. Determine if the following series converges absolutely, conditionally, or not at all.
$$\sum_{n=5}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}-2}.$$
4. Determine if the following series converges absolutely, conditionally, or not at all.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(n)}{n^5}.$$
5. Determine if the following series converges absolutely, conditionally, or not at all.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \cos(n).$$

2. Answer Key

1. Converges.
2. Diverges.
3. Conditionally Converges.
4. Converges absolutely.
5. Diverges.

3. Solutions

1. Consider
$$\sum_{n=1}^{\infty} \left(\frac{\ln(n)}{n}\right)^4.$$
We will do a limit comparison with $\frac{1}{n^2}$. So compute
$$\lim_{n\to\infty} \frac{\left(\frac{\ln(n)}{n}\right)^4}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{(\ln(n))^4}{n^2} \overset{\star}{=} 4 \lim_{n\to\infty} \frac{\ln(n)}{n^2} \overset{\star}{=} 4 \lim_{n\to\infty} \frac{1}{2n^2} = 0$$

Where at $\star$ we applied L'Hôpital's rule. And by limit comparison we have

$$\sum_{n=1}^{\infty}\left(\frac{\ln(n)}{n}\right)^4$$

converges.

2. Consider:

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+2}.$$

We note that

$$\lim n\to\infty(-1)^{n+1}\frac{n}{n+2}\neq 0.$$

Thus, the series diverges.

3. Consider:

$$\sum_{n=5}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}-2}.$$

We see that $\frac{1}{\sqrt{n}-2}$ is decreasing and

$$\lim_{n\to\infty}\frac{1}{\sqrt{n}-2}=0.$$

So by the alternating series test this series converges. Now consider

$$\sum_{n=5}^{\infty}\frac{1}{\sqrt{n}-2}.$$

We note that $\frac{1}{\sqrt{n}}\leq\frac{1}{\sqrt{n}-2}$. And by the p-test we have that $\sum_{n=5}^{\infty}\frac{1}{\sqrt{n}}$. diverges so by comparison we that $\sum_{n=5}^{\infty}\frac{1}{\sqrt{n}-2}$ diverges. Thus,

$$\sum_{n=5}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}-2}.$$

converges conditionally.

4. Consider:

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(n)}{n^5}.$$

Note that $|(-1)^{n+1}\sin(n)|\leq 1$ so $\frac{|(-1)^{n+1}\sin(n)|}{n^5}\leq\frac{1}{n^5}$. And by the p-test we know that $\sum_{n=1}^{\infty}\frac{1}{n^5}$ converges so by comparison we have $\sum_{n=1}^{\infty}\frac{|(-1)^{n+1}\sin(n)|}{n^5}$ converges. Thus,

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(n)}{n^5}$$

converges absolutely.

5. Consider

$$\sum_{n=1}^{\infty}(-1)^{n+1}\cos(n).$$

Note that

$$\lim_{n\to\infty}(-1)^{n+1}\cos(n)$$

does not exist. Thus the series diverges.