MAT 127 HW 15-17

1. Problems

1. Find the Taylor polynomials of degree two approximating the given function centered at the given point.
$$f(x) = \sin(3x), \qquad a = 1$$
2. Find the Taylor polynomials of degree two approximating the given function centered at the given point.
$$f(x) = x^{\frac{1}{3}}, \qquad a = 4$$
3. Find the Taylor polynomials of degree two approximating the given function centered at the given point.
$$f(x) = \ln(x), \qquad a = 2$$
4. Compute
$$\lim_{x\to 0} \frac{\cos x - 1}{2x^2}.$$
5. Compute
$$\lim_{x\to 0} \frac{\tan(x) - x}{13x^3}.$$

2. Answer Key

1. $\sin(3) + 3\cos(3)(x-1) + \frac{-9\sin(3)}{2}(x-1)^2$.
2. $4^{\frac{1}{3}} + \frac{1}{3}4^{\frac{-2}{3}}(x-4) + \frac{-1}{9}4^{\frac{-5}{3}}(x-4)^2$.
3. $\ln(2) + \frac{1}{2}(x-2) + \frac{-1}{8}(x-2)^2$.
4. $\frac{-1}{4}$.
5. $\frac{-1}{39}$.

3. Solutions

1. $f(x) = \sin(3x), a = 1$
$$f(1) = \sin(3), f'(1) = 3\cos(3), f''(1) = -9\sin(3).$$
Thus, the taylor polynomical of degree two is
$$T_2(x) = \sin(3) + 3\cos(3)(x-1) + \frac{-9\sin(3)}{2}(x-1)^2.$$
2. $f(x) = x^{\frac{1}{3}}, a = 4$
$$f(4) = 4^{\frac{1}{3}}, f'(4) = \frac{1}{3}4^{\frac{-2}{3}}, f''(4) = \frac{-2}{9}4^{\frac{-5}{3}}.$$
Thus, the taylor polynomical of degree two is
$$T_2(x) = 4^{\frac{1}{3}} + \frac{1}{3}4^{\frac{-2}{3}}(x-4) + \frac{-1}{9}4^{\frac{-5}{3}}(x-4)^2.$$

3. $f(x) = \ln(x), a = 2$
$$f(2) = \ln(2), f'(2) = \frac{1}{2}, f''(2) = \frac{-1}{4}.$$
Thus, the taylor polynomial of degree two is
$$T_2(x) = \ln(2) + \frac{1}{2}(x-2) + \frac{-1}{8}(x-2)^2.$$
4. Taylor expand $\cos x - 1$ about 0 to get
$$\frac{-x^2}{2} + \frac{x^4}{24} + O(x^8).$$
Now divide this by $2x^2$ to get
$$\frac{-1}{4} + \frac{x^2}{48} + O(x^6).$$
Thus,
$$\lim_{x\to 0} \frac{\cos x - 1}{2x^2} = \lim_{x\to 0} \frac{-1}{4} + \frac{x^2}{48} + O(x^6) = \frac{-1}{4}.$$
5. Taylor expand $\tan x - x$ about 0 to get
$$\frac{-x^3}{3} + \frac{2x^5}{15} + O(x^9).$$
Now divide this by $13x^3$ to get
$$\frac{-1}{39} + \frac{2x^2}{195} + O(x^6).$$
Thus,
$$\lim_{x\to 0} \frac{\tan(x) - x}{13x^3} = \lim_{x\to 0} \frac{-1}{39} + \frac{2x^2}{195} + O(x^6) = \frac{-1}{39}.$$