

MAT 127 HW 11-14

1. PROBLEMS

1. Consider the following power series and find radius of convergence.

$$\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$$

2. Consider the following power series and find radius of convergence.

$$\sum_{n=1}^{\infty} \frac{n^2 x^n}{3^n}$$

3. Consider the following power series and find radius of convergence.

$$\sum_{n=1}^{\infty} 3 \frac{n!}{n^n} x^n$$

4. Consider the following series.

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad g(x) = \sum_{n=0}^{\infty} \frac{x^n}{2^n}.$$

Find $f(x) + g(x)$.

5. Consider the following series.

$$f(x) = \sum_{n=0}^{\infty} \frac{3x^n}{(n+1) \cdot n!}, \quad g(x) = \sum_{n=0}^{\infty} \frac{x^n}{3^n}.$$

Find $2f(x) + 3g(x)$.

2. ANSWER KEY

- 1.
- $\frac{1}{3}$.
- e.
- $\sum_{n=0}^{\infty} \left(\frac{1}{n!} + \frac{1}{2^n} \right) x^n$
- $\sum_{n=0}^{\infty} \left(\frac{6}{(n+1) \cdot n!} + \frac{1}{3^{n-1}} \right) x^n$

3. SOLUTIONS

- 1.

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{(3x)^{n+1}}{n+1}}{\frac{(3x)^n}{n}} \right| = \lim_{n \rightarrow \infty} \frac{3|x| \cdot n}{n+1} = |x|.$$

So $|x| < 1$ And so the radius of convergence is 1.

2.

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{(n+1)^2}{3^{n+1}}}{\frac{n^2}{3^n}} \right| = \lim_{n \rightarrow \infty} \frac{3(n+1)^2}{n^2} = 3.$$

So $|x| < \frac{1}{3}$ And so the radius of convergence is $\frac{1}{3}$.

3.

$$\lim_{n \rightarrow \infty} \left| \frac{3 \frac{(n+1)!}{(n+1)^{n+1}}}{3 \frac{n!}{n^n}} \right| = \lim_{n \rightarrow \infty} (n+1) \cdot \frac{n^n}{(n+1)^n \cdot (n+1)} = \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n = \frac{1}{e}.$$

So $|x| < e$ And so the radius of convergence is e .

4. Add the nth coefficient of each of the series.

$$f(x) + g(x) = \sum_{n=0}^{\infty} \left(\frac{1}{n!} + \frac{1}{2^n} \right) x^n.$$

So by the Ratio Test this converges.

5.

$$2f(x) + 3g(x) = 2 \sum_{n=0}^{\infty} \frac{3x^n}{(n+1) \cdot n!} + 3 \sum_{n=0}^{\infty} \frac{x^n}{3^n} = \sum_{n=0}^{\infty} \left(\frac{6}{(n+1) \cdot n!} + \frac{1}{3^{n-1}} \right) x^n$$