MAT 127 HW 9-10

1. Problems

1. Consider the following series and apply the divergence. If the divergence test does not apply, explain why.
$$\sum_{n=0}^{\infty} \frac{17(n+1)}{n}$$
2. Consider the following series and apply the divergence. If the divergence test does not apply, explain why.
$$\sum_{n=0}^{\infty} e^{\frac{-8}{n^2}}.$$
3. Consider the following series and apply the divergence. If the divergence test does not apply, explain why.
$$\sum_{n=0}^{\infty} \sin(n)$$
4. Consider the following series and apply the ratio test.
$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}$$
5. Consider the following series and apply the ratio test.
$$\sum_{n=1}^{\infty} \frac{(3^n n!)^3}{(3^n)^{3n}}$$

2. Answer Key

1. Does not converge.
2. Does not converge.
3. Does not converge.
4. Converges.
5. Converges.

3. Solutions

1. $\lim_{n\to\infty} \frac{17(n+1)}{n} = 17 \neq 0$. So by divergence test does not converge.
2. $\lim_{n\to\infty} e^{\frac{-8}{n^2}} = 0$. The divergence test does not converge.
3. $\lim_{n\to\infty} \sin(n) = DNE$. So the divergence test does not converge.
4. Consider
$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2}{3^{n+1}}}{\frac{n^2}{3^n}} \right| = \lim_{n\to\infty} \frac{(n+1)^2}{n^2 \cdot 3^n} = 0.$$
So by the Ratio Test this converges.

5. Consider
$$\lim_{n\to\infty}\left|\frac{\frac{(3^{n+1}(n+1)!)^3}{(3^(n+1))^{3(n+1)}}}{\frac{(3^n n!)^3}{(3^n)^{3n}}}\right| = \lim_{n\to\infty}\frac{(3(n+1))^3}{3^{(6n+3)}} = 0.$$
So by the Ratio Test this converges.