

MAT 127 HW 6-8

1. PROBLEM

1. Does the following sequence converge or diverge? If it converge give the what it converges too.

$$\sum_{n=1}^{\infty} \frac{17}{n}.$$

2. Does the following sequence converge or diverge? If it converge give the what it converges too.

$$\sum_{n=0}^{\infty} 7^n.$$

3. Does the following sequence converge or diverge? If it converge give the what it converges too.

$$\sum_{n=0}^{\infty} 23 \cdot \left(\frac{8}{9}\right)^n.$$

4. Does the following sequence converge or diverge? If it converge give the what it converges too.

$$\sum_{n=1}^{\infty} 3^{\frac{1}{n}} - 3^{\frac{1}{n+1}}.$$

5. Does the following sequence converge or diverge? If it converge give the what it converges too.

$$\sum_{n=1}^{\infty} \cos(\pi n) - \cos(\pi n + \pi).$$

2. ANSWER KEY

1. Diverges
2. Diverges
3. Converges. 207.
4. 2.
5. Diverges

3. SOLUTION

1. $\sum_{n=1}^{\infty} \frac{17}{n} = 17 \cdot \sum_{n=1}^{\infty} \frac{1}{n}$ and we identify the harmonic series which is known to diverge.
2. $\sum_{n=0}^{\infty} 7^n$ is a geometric series and $|7| > 1$ so it diverges.
3. $\sum_{n=0}^{\infty} 23 \cdot \left(\frac{8}{9}\right)^n = 23 \cdot \sum_{n=0}^{\infty} \left(\frac{8}{9}\right)^n = 23 \cdot \left(\frac{1}{1-\frac{8}{9}}\right) = 207$

4. $\sum_{n=1}^{\infty} 3^{\frac{1}{n}} - 3^{\frac{1}{n+1}}$. We notice the sum is telescoping and that the k th partial sum is $S_k = 3 - 3^{\frac{1}{k+1}}$. And

$$\lim_{k \rightarrow \infty} S_k = 3 - 1 = 2.$$

5. $\sum_{n=1}^{\infty} \cos(\pi n) - \cos(\pi n + \pi)$. We notice the sum is telescoping and that the k th partial sum is $S_k = -1 - \cos(k + 1)$. And this diverges.