MAT 127 HW 1-5

1. Problems

1. Suppose $\lim_{n\to\infty} a_n = 1$ and $\lim_{n\to\infty} b_n = -1$. Find
$$\lim_{n\to\infty}(5a_n - 8b_n).$$
2. Determine if the sequence
$$a_n = \frac{2}{3^n}, n \geq 1$$
is bounded and whether it is eventually monotone, increasing, or decreasing.
3. Determine if the sequence
$$a_n = \frac{5^n}{n!}, n \geq 1.$$
converges and if it does find the limit.
4. Write the following sum in summation notation.
$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$$
5. Write the first three partial sums S_1, S_2, S_3 for the series having nth term $a_n = 2n+1$ starting with $n = 1$.

2. Answer Key

1. 13.
2. Bounded. Eventually decreasing.
3. Convergent. Limit is 0.
4. $\Sigma_{n=0}^{\infty} \frac{1}{3^n}$
5. $S_1 = 3$, $S_2 = 8$, $S_3 = 15$

3. Solutions

1. $\lim_{n\to\infty}(5a_n - 8b_n) = 5\lim_{n\to\infty} a_n - 8\lim_{n\to\infty} b_n = 5 + 8 = 13.$
2. Since for all $n \geq 1$ we have that $0 \geq 2 \leq 3^n$, we see that $0 \leq a_n \leq 1$. So the sequence is bounded. Now consider
$$\frac{a_{n+1}}{a_n} = \frac{2\cdot 3^n}{2\cdot 3^{n+1}} = \frac{1}{3} < 1.$$
Thus, $a_{n+1} < a_n$.
3.
$$a_{n+1} = \frac{5^{n+1}}{(n+1)!} = \frac{5}{n+1}\cdot\frac{5^n}{n!} = \frac{5}{n+1}\cdot a_n.$$
Thus, a_n is decreasing when $n \geq 4$. And we note $a_n \geq 0$ for all n. Thus, a_n is convergent and call the limit L. Moreover,
$$a_{n+1} = \frac{5}{n+1}\cdot a_n.$$

So

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{5}{n+1} \cdot a_n.$$

Thus,

$$L = 0 \cdot L.$$

And we conclude $L = 0$.

4. We notice that

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots = \frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \cdots$$

Thus, this is the sum of the terms of sequence $a_n = \frac{1}{3^n}$ where $n \geq 0$. Therefore, in summation notation we have $\Sigma_{n=0}^{\infty} \frac{1}{3^n}$.

5. $S_1 = a_1 = 3$, $S_2 = a_1 + a_2 = 3 + 5 = 8$, $S_3 = a_1 + a_2 + a_3 = 3 + 5 + 7 = 15$