MAT 126 HW 22-23

1. Problems

1. Find the length of the curve $y = \frac{2}{3}(x^2+1)^{3/2}$ over the interval $[1,4]$.
2. Find the length of the curve $y = 2(x-1)^{3/2}$ over the interval $[1,5]$.
3. Find the length of the curve $y = \ln(\cos x)$ over the interval $[0.\pi/4]$.
4. Find the average value of the function $f(x) = 4x^3 + 1$ on the interval $[-1,3]$
5. Find the average value of the function $g(x) = 4e^{3x+1}$ on the interval $[0,2]$.

2. Answer Key

1. 45
2. $\frac{2}{27}\left(37^{3/2}-1\right)$
3. $\ln\left(\sqrt{2}+1\right)$
4. 21
5. $\frac{2}{3}\left(e^{5}-e\right)$

3. Solutions

1. Differentiating gives
$$\frac{dy}{dx} = 2x\sqrt{x^2+1}.$$
It follows that $1+(dy/dx)^2 = 1+4x^2(x^2+1) = 4x^4+4x^2+1 = (2x^2+1)^2$. Hence the arclength is
$$\int_1^4 \sqrt{1+(dy/dx)^2}\;dx = \int_1^4 \sqrt{(2x^2+1)^2}\;dx = \int_1^4 2x^2+1\;dx = \frac{2x^3}{3}+x\,\bigg|_1^4 = 45.$$

2. Differentiating gives
$$\frac{dy}{dx} = 3\sqrt{x-1}.$$
It follows that $1+(dy/dx)^2 = 1+9x-9 = 9x-8$. Hence the arclength is
$$\int_1^5 \sqrt{1+(dy/dx)^2}\;dx = \int_1^5 \sqrt{9x-8}\;dx = \frac{2}{27}(9x-8)^{3/2}\,\bigg|_1^5 = \frac{2}{27}\left(37^{3/2}-1\right).$$

3. Differentiating gives
$$\frac{dy}{dx} = -\tan x.$$
It follows that $1+(dy/dx)^2 = 1+\tan^2 x = \sec^2 x$. Hence the arclength is
$$\begin{aligned}\int_0^{\pi/4} \sqrt{1+(dy/dx)^2}\;dx &= \int_0^{\pi/4} \sqrt{\sec^2 x}\;dx \\ &= \int_0^{\pi/4} \sec x\;dx \\ &= \ln|\sec x+\tan x|\,\bigg|_0^{\pi/4} \\ &= \ln|\sqrt{2}+1| - \ln|1+0| \\ &= \ln\left(\sqrt{2}+1\right).\end{aligned}$$

4.
$$\frac{1}{3-(-1)}\int_{-1}^3 4x^3+1\;dx = \frac{1}{4}\left(x^4+x\right)\bigg|_{-1}^3 = \frac{84}{4} = 21$$

5.
$$\frac{1}{2-0}\int_0^2 4e^{3x+1}\;dx = \frac{2}{3}e^{2x+1}\,\bigg|_0^2 = \frac{2}{3}\left(e^5-e\right).$$