MAT 126 HW 21

1. Problems

1. Calculate the volume obtained by revolving the region bounded by the graphs of $f(x) = x^2 + 2x + 1$ and $g(x) = 3x + 1$ about the x-axis.
2. Calculate the volume obtained by revolving the same region as in the previous problem about the y-axis.
3. Calculate the volume obtained by revolving the region bounded by the graph of $f(x) = \cos x$ and the x- and y-axes about the y-axis.
4. Calculate the volume obtained by revolving the same region as in the previous problem about the x-axis.
5. Calculate the volume obtained by revolving the same region as in the previous problem about the line $x = -1$.

2. Answer Key

1. $$\frac{4\pi}{5}$$
2. $$\frac{\pi}{6}$$
3. $\pi^2 - 2\pi$
4. $$\frac{\pi^2}{4}$$
5. π^2

3. Solutions

1. Graphing, we see that $g(x)$ lies above $f(x)$. To solve for the endpoints of our interval of integration, we write $x^2+2x+1=3x+1$. Solving for x yields $x=0$ and $x=1$. Using the washer method, the volume is given by
$$\begin{aligned}\pi\int_0^1 (2x+1)^2-(x^2+2x+1)^2\,dx &= \pi\int_0^1 -x^4-4x^3+3x^2+2x\,dx\\ &= \pi\left(-\frac{1}{5}x^5-x^4+x^3+x^2\right)\Big|_0^1 = \frac{4\pi}{5}\end{aligned}$$
2. Now we use the shell method. We have
$$\begin{aligned}2\pi\int_0^1 x(3x+1-x^2-2x-1)\,dx &= 2\pi\int_0^1 -x^3+x^2\,dx\\ &= 2\pi\left(-\frac{1}{4}x^4+\frac{1}{3}x^3\right)\Big|_0^1 = \frac{\pi}{6}\end{aligned}$$
3. The region we are considering is the one bounded by the graph of $\cos x$ and which lies in the first quadrant of the plane. The left and rightmost endpoints of this are at $x=0$ and $x=\pi/2$ respectively. Using the shell method, we write $2\pi\int_0^{\pi/2} x\cos x\,dx$. To solve this we use integration by parts with $u=x$ and $dv=\cos x\,dx$. This yields
$$\begin{aligned}2\pi\int_0^{\pi/2} x\cos x\,dx &= 2\pi\left(x\sin x\,\Big|_0^{\pi/2}-\int_0^{\pi/2}\sin x\,dx\right)\\ &= 2\pi\left(x\sin x+\cos x\right)\Big|_0^{\pi/2} = \pi^2-2\pi\end{aligned}$$
4. Using the washer method now, the desired volume is
$$\pi\int_0^{\pi/2}\cos^2 x\,dx = \frac{\pi}{2}\int_0^{\pi/2} 1+\cos 2x\,dx = \frac{\pi}{2}\left(x+\frac{1}{2}\sin 2x\right)\Big|_0^{\pi/2} = \frac{\pi^2}{4}.$$
5. The setup is much like in problem 3, however this time the radius of our shells is $x+1$ instead of x, so the desired volume is
$$\begin{aligned}2\pi\int_0^{\pi/2}(x+1)\cos x\,dx &= 2\pi\int_0^{\pi/2} x\cos x\,dx + 2\pi\int_0^{\pi/2}\cos x\,dx\\ &= \pi^2-2\pi+(2\pi\sin x)\Big|_0^{\pi/2} = \pi^2.\end{aligned}$$