MAT 126 HW 12-14

1. Problems

1. Compute the following indefinite integral:
$$\int \frac{17x - 53}{x^2 - 2x - 15}\, dx$$
2. Compute the following indefinite integral:
$$\int \frac{3x + 9}{(x + 1)^2}\, dx$$
3. Compute the following indefinite integral:
$$\int \frac{x^2 + 2x - 4}{x + 3}\, dx$$
4. Compute the following indefinite integral:
$$\int \frac{5x - 3x + 1}{x(x^2 + 1)}\, dx$$
5. Determine whether the following improper integral converges or diverges. If it converges, compute the result:
$$\int_0^\infty \cos(4x + 5)\, dx$$

2. Answer Key

1.
$$4 \ln |x-5| + 13 \ln |x+3| + C$$

2.
$$3 \ln |x+1| - \frac{6}{x+1} + C$$

3.
$$\frac{1}{2}x^2 - x - \ln |x+3| + C$$

4.
$$4 \ln |x| + \frac{1}{2} \ln(x^2+1) - 3 \arctan x + C$$

5. The integral diverges

3. Solutions

1. Factoring the denominator gives $x^2 - 2x - 15 = (x-5)(x+3)$. Performing partial fraction decomposition, we write
$$\frac{17x - 53}{x^2 - 2x - 15} = \frac{A}{x-5} + \frac{B}{x+3}$$
and we find that $A = 4$ and $B = 13$. Hence
$$\int \frac{17x-53}{x^2-2x-15}\,dx = \int \frac{4}{x-5} + \frac{13}{x+3}\,dx = 4\ln|x-5| + 13\ln|x+3| + C.$$

2. Performing partial fraction decomposition, we write
$$\frac{3x+9}{(x+1)^2} = \frac{A}{x+1} + \frac{Bx+C}{(x+1)^2}$$
and we find that $A = 3$, $B = 0$, and $C = 6$. Hence
$$\int \frac{3x+9}{(x+1)^2}\,dx = \int \frac{3}{x+1} + \frac{6}{(x+1)^2}\,dx = 3\ln|x+1| - \frac{6}{x+1} + C$$

3. For this we must do polynomial division. Doing this yields
$$\frac{x^2+2x-4}{x+3} = x - 1 - \frac{1}{x+3}.$$
This gives
$$\int \frac{x^2+2x-4}{x+3}\,dx = \int x - 1 - \frac{1}{x+3}\,dx = \frac{1}{2}x^2 - x - \ln|x+3| + C.$$

4. Performing partial fraction decomposition, we write
$$\frac{5x-3x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$
and we find that $A = 4$, $B = 1$, and $C = -3$. This gives us
$$\begin{aligned}\int \frac{5x-3x+1}{x(x^2+1)}\,dx &= \int \frac{4}{x} + \frac{x-3}{x^2+1}\,dx \\ &= \int \frac{4}{x}\,dx + \int \frac{x}{x^2+1}\,dx - \int \frac{3}{x^2+1}\,dx.\end{aligned}$$
On the right hand side, the first integral gives us $4\ln|x|$. The second, after performing a substitution with $u = x^2+1$ gives us $\frac{1}{2}\ln(x^2+1)$. The third integrates to $-3\arctan x$. Hence the final answer is
$$\int \frac{5x-3x+1}{x(x^2+1)}\,dx = 4\ln|x| + \frac{1}{2}\ln(x^2+1) - 3\arctan x + C$$

5. By definition, the improper integral is given by
$$\int_0^\infty \cos(4x+5)\,dx = \lim_{c\to\infty}\int_0^c \cos(4x+5)\,dx.$$
The antiderivative of the integrand is $\frac{1}{4}\sin(4x+5)$, so this then becomes
$$\int_0^\infty \cos(4x+5)\,dx = \lim_{c\to\infty}\int_0^c \cos(4x+5)\,dx = \lim_{c\to\infty}\frac{1}{5}\left(\sin(4c+5) - \sin(4\cdot 0+5)\right).$$

Now, since $\sin(4 \cdot 0 + 5) = \sin 5$ is a real number and $\lim_{c\to\infty} \sin(4c + 5)$ does not exist due to the oscillatory nature of sine, this limit does not exist. Hence the integral diverges.