MAT 126 HW 9

1. Problems

1. Compute the following indefinite integral using integration by parts:
$$\int x\cos(5x+3)\,dx$$
2. Compute the following indefinite integral using integration by parts:
$$\int x^2e^{-2x}\,dx$$
3. Compute the following indefinite integral using integration by parts:
$$\int t^7\sin(3t^4)\,dt$$
(HINT: First perform a u-substitution, then do integration by parts).
4. Compute the following definite integral using integration by parts:
$$\int_0^{\pi} e^x\sin x\,dx$$
5. Compute the following indefinite integral using integration by parts:
$$\int t^{99}\ln t\,dt$$

2. Answer Key

1.
$$\frac{x\sin(5x+3)}{5}+\frac{\cos(5x+3)}{25}+C$$

2.
$$-\frac{1}{2}\left(x^2+x+\frac{1}{2}\right)e^{-2x}+C$$

3.
$$\frac{1}{36}(3t^4\cos(3t^4)+\sin(3t^4))$$

4.
$$\frac{e^\pi+1}{2}$$

5.
$$\frac{t^{100}\ln t}{100}-\frac{t^{100}}{10000}+C$$

3. Solutions

1. Letting $u = x$ and $dv = \cos(5x+3)\ dx$ gives us $du = dx$ and $v = \frac{1}{5}\sin(5x+3)\ dx$. Hence we get

$$\begin{aligned}\int x\cos(5x+3)\ dx &= \frac{x\sin(5x+3)}{5} - \frac{1}{5}\int \sin(5x+3)\\ &= \frac{x\sin(5x+3)}{5} + \frac{\cos(5x+3)}{25} + C\end{aligned}$$

2. We have to do integration by parts twice. Letting $u = x^2$ and $dv = e^{-2x}\ dx$ gives us $du = 2x\ dx$ and $v = -\frac{1}{2}e^{-2x}$. This gives us

$$\int x^2e^{-2x}\ dx = -\frac{1}{2}x^2e^{-2x} + \int xe^{-2x}\ dx.$$

Again performing integration by parts with $u = x$ and $dv = e^{-2x}\ dx$ gives us

$$\int xe^{-2x}\ dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\ dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C.$$

Putting this all together, we get

$$\begin{aligned}\int x^2e^{-2x}\ dx &= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C\\ &= -\frac{1}{2}\left(x^2 + x + \frac{1}{2}\right)e^{-2x} + C.\end{aligned}$$

3. First we perform a substitution (we'll use the letter x to not confuse our new variable with the u in integration by parts). Let $x = 3t^4$ so that $dx = 12t^3$. We get

$$\int t^7\sin(3t^4)\ dt = \int \frac{1}{12}t^4\sin x\ dx.$$

Notice that $t^4 = x/3$ so this integral equals $(1/36)\int x\sin x\ dx$. Now we can perform integration by parts with $u = x$ and $dv = \sin x\ dx$ which gives us

$$\int \frac{1}{12}t^4\sin x\ dx = \frac{1}{36}\left(-x\cos x + \int \cos x\right) = \frac{1}{36}\left(-x\cos x + \sin x\right) + C.$$

Substituting back into the variable t we finally get

$$\int t^7\sin(3t^4)\ dt = \frac{1}{36}\left(-3t^4\cos(3t^4) + \sin(3t^4)\right) + C.$$

4. This one needs a trick that was shown in the lecture. Let $I = \int e^x\sin x\ dx$. Doing integration by parts with $u = \sin x$ and $dv = e^x\ dx$ gives us

$$I = e^x\sin x - \int e^x\cos x\ dx.$$

Again performing integration by parts in the same way with $u = \cos x$ and $dv = e^x\ dx$ gives us

$$I = e^x\sin x - \int e^x\cos x\ dx = e^x\sin x - e^x\cos x - I.$$

Adding I to both sides and then dividing by 2 yields

$$I = \frac{e^x\sin x - e^x\cos x}{2}$$

Now we substitute at $x = 0$ and $x = \pi$

$$\int_0^\pi e^x \sin x \, dx = \left. \frac{e^x \sin x - e^x \cos x}{2} \right|_0^\pi = \frac{e^\pi + 1}{2}$$

5. Let $u = \ln t$ and $dv = t^{99} \, dt$ so that $du = \frac{dt}{t}$ and $v = \frac{1}{100} t^{100}$. Then we get

$$\int t^{99} \ln t \, dt = \frac{t^{100} \ln t}{100} - \frac{1}{100} \int t^{99} \, dt = \frac{t^{100} \ln t}{100} - \frac{t^{100}}{10000} + C$$