MAT126 Homework 6-8

Problems

1. Evaluate the following indefinite integrals:
   (a) $\int sec^2(5x)dx$
   (b) $\int e^{3x}$
2. Evaluate the following indefinite integrals:
   (a) $\int (8x-12)\left(4x^2-12x\right)^4 dx$
   (b) $\int 3x^{-4}\left(2+4x^{-3}\right)^{-7} dx$
3. Evaluate the following indefinite integrals:
   (a) $\int \frac{2x}{1+4x^2}\, dx$
   (b) $\int \frac{2}{1+4x^2}\, dx$
4. Evaulate the following indefinite integrals:
   (a) $\int \frac{4x+3}{4x^2+6x-1}\, dx$
   (b) $\int \frac{4}{(6+9x)^5} + \frac{13}{6+9x}\, dx$
5. Evaluate the following indefinite integrals:
   (a) $\int \tan(x)dx$
   (b) $\int 4\sin(2x)\cos(2x)\sqrt{\cos^2(2x)-5}\, dx$

Answer Key

1. (a) $\int \sec^2(5x)dx = \frac{1}{5}\tan(5x) + C$

   (b) $\int e^{3x}dx = \frac{1}{3}e^{3x} + C$

2. (a) $\int (8x-12)\left(4x^2-12x\right)^4 dx = \frac{1}{5}(4x^2-12x)^5 + C$

   (b) $\int 3x^{-4}\left(2+4x^{-3}\right)^{-7} dx = \frac{1}{24}(2+4x^{-3})^{-6} + C$

3. (a) $\int \frac{2x}{1+4x^2}\,dx = \frac{1}{4}\ln(1+4x^2) + C$

   (b) $\int \frac{2}{1+4x^2}\,dx = \arctan(2x) + C$

4. (a) $\int \frac{4x+3}{4x^2+6x-1}\,dx = \frac{1}{2}\ln|4x^2+6x-1| + C$

   (b) $\int \frac{4}{(9+6x)^5} + \frac{13}{9+6x}\,dx = -\frac{1}{6}(9+6x)^{-4} + \frac{13}{6}\ln|9+6x| + C$

5. (a) $\int \tan(x)dx = -\ln|cos(x)| + C$

   (b) $\int 4\sin(2x)\cos(2x)\sqrt{\cos^2(2x)-5}\,dx = -\frac{2}{3}(\cos^2(2x)-5)^{\frac{3}{2}} + C$

Solutions

1. (a) Let $u = 5x$. Then $du = 5dx$ and it follows that

$$\int \sec^2(5x)dx = \frac{1}{5}\int sec^2(u)du = \frac{1}{5}\tan(u) + C = \frac{1}{5}\tan(5x) + C$$

   (b) Let $u = 3x$. Then $du = 3dx$ and it follows that

$$\int e^{3x}dx = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + C = \frac{1}{3}e^{3x} + C$$

2. (a) Let $u = 4x^2 - 12x$. Then $du = (8x - 12)dx$ and it follows that

$$\begin{aligned}\int (8x-12)\left(4x^2-12x\right)^4 dx &= \int u^4 du \\ &= \frac{1}{5}u^5 + C \\ &= \frac{1}{5}(4x^2-12x)^5 + C\end{aligned}$$

   (b) Let $u = 2 + 4x^{-3}$. Then $du = (-12x^{-4})dx$ and it follows that

$$\begin{aligned}\int 3x^{-4}\left(2+4x^{-3}\right)^{-7} dx &= -\frac{1}{4}\int u^{-7}du \\ &= \frac{1}{24}u^{-6} + C \\ &= \frac{1}{24}(2+4x^{-3})^{-6} + C\end{aligned}$$

3. (a) Let $u = 1 + 4x^2$. Then $du = 8xdx$ and it follows that

$$\begin{aligned}\int \frac{2x}{1+4x^2}dx &= \frac{1}{4}\int \frac{1}{u}du \\ &= \frac{1}{4}\ln(u)du + C \\ &= \frac{1}{4}\ln(1+4x^2) + C\end{aligned}$$

   (b) Let $u = 2x$. Then $du = 2dx$ and it follows that

$$\int \frac{2}{1+4x^2}dx = \int \frac{1}{1+u^2}du \quad = \arctan(u) + C = \arctan(2x) + C$$

4. (a) Let $u = 4x^2 + 6x - 1$. Then $du = (8x + 6)dx$ and it follows that

$$\begin{aligned}\int \frac{4x+3}{4x^2+6x-1}dx &= \frac{1}{2}\int \frac{1}{u}du \\ &= \frac{1}{2}\ln|u| + C \\ &= \frac{1}{2}\ln|4x^2+6x-1| + C\end{aligned}$$

(b) Let $u = 9 + 6x$. Then $du = 6dx$ and it follows that

$$\begin{aligned}\int \frac{4}{(9+6x)^5} + \frac{13}{9+6x}\,dx &= \frac{2}{3}\int u^{-5}du + \frac{13}{6}\int \frac{1}{u}du \\ &= -\frac{1}{6}u^{-4} + \frac{13}{6}\ln|u| + C \\ &= -\frac{1}{6}(9+6x)^{-4} + \frac{13}{6}\ln|9+6x| + C\end{aligned}$$

5. (a) Notice that $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Let $u = \cos(x)$. Then $du = -\sin(x)dx$ and it follows that

$$\begin{aligned}\int \tan(x)dx &= \int \frac{\sin(x)}{\cos(x)}dx \\ &= -\int \frac{1}{u}du \\ &= -\ln|u| + C \\ &= -\ln|\cos(x)| + C\end{aligned}$$

(b) Let $u = \cos^2(2x) - 5$. Then $du = -4\cos(2x)\sin(2x)$ and it follows that

$$\begin{aligned}\int 4\sin(2x)\cos(2x)\sqrt{\cos^2(2x) - 5}\,dx &= -\int u^{\frac{1}{2}}du \\ &= -\frac{2}{3}u^{\frac{3}{2}} + C \\ &= -\frac{2}{3}(\cos^2(2x) - 5)^{\frac{3}{2}} + C\end{aligned}$$