MAT126 Homework 1

Problems

1. Find the derivative of $f(x) = x^7 + \cos(3x) + e^{x^2}$.

2. Find the derivative of $f(x) = \dfrac{x^2 e^x}{\tan(\pi x)}$.

3. If $f(3) = 10$, $f'(3) = 7$, $g(3) = 3$, $g'(3) = -4$, find

   (a) $(fg)'(3)$

   (b) $(\frac{f}{g})'(3)$

   (c) $(f \circ g)'(3)$

4. Find the general antiderivative of $f(x) = x^{\frac{1}{3}} + \dfrac{1}{x} + \sec(x)\tan(x) + 2e^x$.

5. Find the antiderivative of $f(x) = \dfrac{x^2 + 3x + 27}{x^2}$ if $F(1) = 0$.

6. If $f(x) = \dfrac{1}{1 + x^2} + \sec^2(x)$ and $F(0) = \pi$, find $F(\pi/4)$.

Answer Key

1. $f'(x) = 7x^6 - 3\sin(3x) + 2xe^{x^2}$.

2. $f'(x) = \dfrac{(2xe^x + x^2e^x)tan(\pi x) - (x^2e^x)\pi\sec^2(\pi x)}{tan^2(\pi x)}$.

3. (a) $(fg)'(3) = 58$.

   (b) $\left(\dfrac{f}{g}\right)'(3) = \dfrac{82}{9}$.

   (c) $(f \circ g)'(3) = -40$.

4. $F(x) = \dfrac{3}{4}x^{\frac{4}{3}} + \ln(x) + \sec(x) + 2e^x + C$.

5. $F(x) = x + 3\ln(x) - \dfrac{27}{x} + 26$.

6. $F(\dfrac{\pi}{4}) = \arctan(\dfrac{\pi}{4}) + \dfrac{\sqrt{2}}{2} + \pi$.

Solutions

1. $f'(x) = 7x^6 - 3\sin(3x) + 2xe^{x^2}$.

2. $f'(x) = \dfrac{(2xe^x + x^2e^x)\tan(\pi x) - (x^2e^x)\pi\sec^2(\pi x)}{\tan^2(\pi x)}$.

3. (a) By the product rule,
$$(fg)'(3) = f'(3)g(3) + f(3)g'(3) = 10(7) + 3(-4) = 70 - 12 = 58.$$
   (b) By the quotient rule,
$$\left(\frac{f}{g}\right)'(3) = \frac{f'(3)g(3) - f(3)g'(3)}{g(3)^2} = \frac{10(7) - 3(-4)}{3^2} = \frac{82}{9}.$$
   (c) By the chain rule,
$$(f \circ g)'(3) = f(g(3))g'(3) = f(3) * (-4) = 10(-4) = -40.$$

4. $F(x) = \dfrac{3}{4}x^{\frac{4}{3}} + \ln(x) + \sec(x) + 2e^x + C$.

5. Notice that $f(x) = 1 + \frac{3}{x} + 27x^{-2}$. Using our basic antiderivatives, we see that the general antiderivative of $f(x)$ is
$$F(x) = x + 3\ln(x) - \frac{27}{x} + C.$$
Since $F(1) = 0$,
$$0 = 1 + 3\ln(1) - 27 + C = -26 + C.$$
Solving for C gives $C = 26$. It follows that the antiderivative satisfying $F(1) = 0$ is
$$F(x) = x + 3\ln(x) - \frac{27}{x} + 26.$$

6. Using our basic antiderivatives, we see that the general antiderivative of $f(x)$ is
$$F(x) = \arctan(x) + \tan(x) + C.$$
Since $F(0) = \pi$,
$$\pi = \arctan(0) + \tan(0) + C = C.$$
It follows that $C = \pi$ and the antiderivative satisfying $F(0) = \pi$ is
$$F(x) = \arctan(x) + \tan(x) + \pi.$$
It follows that
$$F\left(\frac{\pi}{4}\right) = \arctan\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right) + \pi = \arctan\left(\frac{\pi}{4}\right) + \frac{\sqrt{2}}{2} + \pi.$$