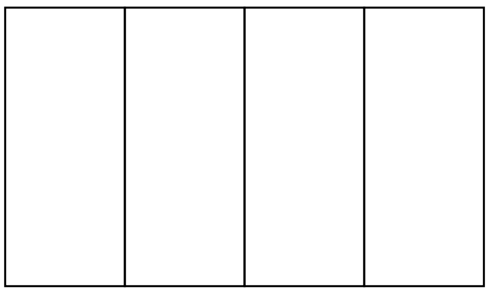
MAT125 Homework for Lectures 24-26

July 13, 2021

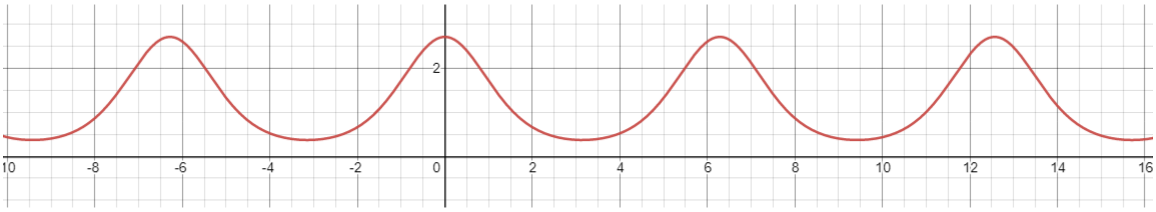
1 Problems

1. Find the antiderivative of $f(x) = x^{2020} - 3\sin(5x)$.
2. Find the antiderivative of the antiderivative of the antiderivative of $f(x) = x^7 - x^5 + \frac{1}{x^3}$.
3. Find the derivative of $y = \tan^{-1}(x)$ using implicit differentiation.
4. Find the antiderivative of $g(u) = (u^2 + 1)^{-1}$.
5. Graph $f(x) = e^{\cos(x)}$ on the interval $[-3\pi, 5\pi]$ and label the critical points/values in (x, y) form.
6. A corral consists of 4 identical rectangular pens that share fences as in the picture. If 400 m of fence is available, what are the dimensions of the corral and the maximum possible area of the corral?



2 Answer Key

1. $F(x) = \frac{x^{2021}}{2021} + \frac{3}{5}\cos(5x) + C$.
2. $F(x) = \frac{x^{10}}{720} - \frac{x^8}{336} + \frac{1}{2}\ln(x) + C$
3. $\frac{dy}{dx} = \frac{1}{1+x^2}$.
4. $G(u) = \tan^{-1}(u) + C$
5. The maxima are of the form $(2\pi k, e)$ where $k = -1, 0, 1, 2$ and the minima are of the form $((2j+1)\pi, 1/e)$ where $j = -2, -1, 0, 1, 2$.



6. Dimensions: 100×40 m; maximum area: 4000 sq m.

3 Solution

1. Just use the power rule for the first term and think about the derivatives of $\sin(x)$ and $\cos(x)$ for the second term. Those derivatives (up to constants) have a periodic behavior.

2. Apply power rule 3 times. The last time, recall that $\frac{d}{dx}\ln(x) = 1/x$.

3. Rewrite the equation as $\tan y = x$. Then implicit differentiation gives $\sec^2(y)\frac{dy}{dx} = 1$ so $\frac{dy}{dx} = \cos^2(y)$. We may reinterpret $\tan(y) = x$ as saying that the tangent of the angle y of a right triangle is $x/1$; i.e. the opposite over the adjacent. Hence, the hypotenuse has length $\sqrt{1+x^2}$. The cosine of the angle is adjacent over hypotenuse; squaring this, we get $\cos^2(y) = \frac{1}{1+x^2}$.

4. Question 3 basically gives the answer: $G(u) = \tan^{-1}(u) + C$.

5. $f'(x) = -\sin(x)e^{\cos(x)}$; this equals zero whenever $\sin(x) = 0$ which is at multiples of π. One can see that even multiples of π are the maxima since $\cos(2\pi k) = 1$ and the odd multiples of π are the minima since $\cos((2k+1)\pi) = -1$. The graph is always above the x-axis; the maximum values are e, the minimum values are $1/e$.

   One can figure out the intervals on which f is increasing/decreasing by studying the derivative but it only depends on the derivative of $\cos(x)$.

6. Let x be the width of one of the rectangular enclosures and y the height. So the total area of the corral is $A = 4xy$ and the amount of fencing needed is described by $8x + 5y$ and is constrained to $8x + 5y = 400$. So then $y = (400 - 8x)/5$ and $A = 4x(400 - 8x)/5$. Then $dA/dx = (1600 - 64x)/5$. Setting this equal to zero, we find the critical point is at $x = 25$ m. This is a maximum since the quadratic function $A(x)$ is concave down. And $y = 40$ m. Hence, the dimensions are 100×40 m and the maximum area is 4000 sq m.