1

Compute the following limit:

$$\lim_{x\to 5}(x^2+5)(x^2-5)$$

2

Find the limit as $t \to 0^-$ of the following function:

$$f(t) = \begin{cases} t+2 & t>0 \\ -3 & t \leq 0 \end{cases}$$

Is this function continuous? If so, why? If not, where is it discontinuous and why?

3

Compute the limit:

$$\lim_{x\to -1}\frac{x^2+2x+1}{x+1}$$

4

Calculate the following limits:

1. $\lim_{x\to\infty}\frac{6x^2+2x+3}{2x^2+x-2}$
2. $\lim_{x\to-\infty}\frac{x^4+1}{x^8+1}$
3. $\lim_{x\to-\infty} e^{-x^2}$

5

Find the value of a that makes the following function continuous on all of $(-\infty,\infty)$:

$$h(x) = \begin{cases} x^2+5 & x>-2 \\ ax-1 & x \leq -2 \end{cases}$$

Answer Key

1. The limit is 600.

2. The limit is -3 and the function is discontinuous at the point t=-3.

3. The limit is 0.

4. (i) 3, (ii) 0, (iii) 0.

5. The required value is a=-5.

Solutions

1. Using the product rule, we see that:

$$\lim_{x\to 5}(x^2+5)(x^2-5) = \lim_{x\to 5}(x^2+5)\lim_{x\to 5}(x^2-5) = (5^2+5)(5^2-5) = 30\times 20 = 600$$

The product rule is justified because both limits $\lim_{x\to 5}(x^2+5)$ and $\lim_{x\to 5}(x^2-5)$ exist.

2. We see that $\lim_{t\to 0^-} f(t) = \lim_{t\to 0^-}(-3) = -3$, since $f(t)$ is constant for all $t \leq 0$. On the other hand, $\lim_{t\to 0^+} f(t) = \lim_{t\to 0^+}(t+2) = 0+2 = 2$. Since these two limits fail to agree ($-3 \neq 2$), we observe that $f(t)$ is discontinuous at $t = -3$.

3. Factoring $x^2+2x+1 = (x+1)^2$ allows us to make a cancellation with the denominator and therefore:

$$\lim_{x\to -1}\frac{x^2+2x+1}{x+1} = \lim_{x\to -1}\frac{(x+1)^2}{x+1} = \lim_{x\to -1}(x+1) = -1+1 = 0$$

Notice that the limit could not have been evaluated by direct application of the quotient rule as this would have yielded the indeterminate form $0/0$.

4. The first two limits are evaluated using the rules for rational functions. Since the leading terms $6x^2$ and $2x^2$ in the numerator and denominator, respectively, have the same order, we have:

$$\lim_{x\to\infty}\frac{6x^2+2x+3}{2x^2+x-2} = \lim_{x\to\infty}\frac{6x^2}{2x^2} = \lim_{x\to\infty}\frac{6}{2} = \frac{6}{2} = 3$$

Since the leading term x^8 in the denominator has a higher order than the leading term x^4 in the numerator, we see that:

$$\lim_{x\to-\infty}\frac{x^4+1}{x^8+1} = \lim_{x\to-\infty}\frac{x^4}{x^8} = \lim_{x\to-\infty}\frac{1}{x^4} = 0$$

There are several ways to do the final part of this problem. One is to observe that for $x < 0$, as x tends towards $-\infty$, the function e^{-x^2} is monotonically decreasing and bounded below by 0, so its limit must be 0. Another is to use the limit laws for exponential functions, observing that $\lim_{x\to-\infty}(-x^2) = -\infty$.

5. To make the function $h(x)$ continuous, we must have that $\lim_{x\to-2^+} h(x) = \lim_{x\to-2^-} h(x)$. Now:

$$\lim_{x\to-2^+} h(x) = \lim_{x\to-2^+}(x^2+5) = (-2)^2+5 = 4+5 = 9$$

$$\lim_{x\to-2^-} h(x) = \lim_{x\to-2^-}(ax-1) = a(-2)-1 = -2a-1$$

Hence, we must solve the linear equation:

$$-2a-1 = 9 \;\Rightarrow\; -2a = 10 \;\Rightarrow\; a = -5$$