

1 Problems

Exercise 1. The half life of an element is 4 hours. If there are 200 grams originally, how much will there be after 20 hours?

Exercise 2. If you have 100 grams of an element originally and after 4 hours you have 50 grams, how many will you have after 8 hours?

Exercise 3. Solve $\log_2(2x + 1) - \log_2(x + 1) = 4$ for x if there is a solution. Otherwise say no solution.

Exercise 4. Solve $8^{x+1} = 4^{x+2}$ for x .

Exercise 5. Solve $8^{x+1} = 11$ for x in terms of log.

2 Answer key

Exercise 1. $\frac{25}{4}$ grams

Exercise 2. 25 grams

Exercise 3. No solution.

Exercise 4. $x = 0$.

Exercise 5. $x = \log_8 11 - 1$.

3 Solutions

Exercise 1. We use the formula $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{K}}$ where K is the half life. Then $A_0 = 200$ and $K = 4$ and $t = 20$. So $A(20) = 200\left(\frac{1}{2}\right)^{\frac{20}{4}} = 200 \cdot \frac{1}{2^5} = \frac{200}{32} = \frac{25}{4}$.

Exercise 2. The general exponential formula is $y = ab^x$ where a is the initial amount and b is the growth or decay rate. So we have $50 = 100b^4$ so $b = \frac{1}{2}^{\frac{1}{4}}$. Then plugging in $x = 8$ for 8 hours later into the formula, we get $y = 100\left(\frac{1}{2}\right)^{\frac{8}{4}} = 100\left(\frac{1}{2}\right)^2 = \frac{100}{4} = 25$.

Exercise 3. The whole expression is equal to $\log_2\left(\frac{2x+1}{x+1}\right) = 4$ so $2^4 = \frac{2x+1}{x+1}$. This is equal to $16(x+1) = 2x+1$ so $x = -\frac{15}{14}$. But plugging in x into the expressions give negative numbers, and log of a negative number is not well defined.

Exercise 4. The left hand side is 4^{2x+2} so we need to solve $2x+2 = x+2$. This gives $x = 0$.

Exercise 5. $8^{x+1} = 11$ so take log on both sides to get $\log_8(8^{x+1}) = \log_8 11$. Then the left hand side is equal to $x+1$ so we get $x+1 = \log_8 11$. So $x = \log_8 11 - 1$.