Lecture 27
MAT 123

More Solving Equations...

① $e^{2x} - 6e^x - 7 = 0$

let $w = e^x$

$w^2 - 6w - 7 = 0$

$(w-7)(w+1) = 0$

$w - 7 = 0$ | $w + 1 = 0$

$w = 7$ | $w = -1$

$e^x = 7$ | $e^x = -1$

$x = \ln 7$ | No solution

Note:
$e^{2x} = (e^x)^2$

② $e^x - 8e^{-x} = 7$

$\left(e^x - \frac{8}{e^{x}} = 7\right) \cdot e^x$

Note: $e^{-x} = \frac{1}{e^x}$

$e^{2x} - 8 = 7e^x$

$e^{2x} - 7e^x - 8 = 0$

let $y = e^x$

$y^2 - 7y - 8 = 0$

$(y-8)(y+1) = 0$

$y - 8 = 0$ | $y + 1 = 0$

$y = 8$ | $y = -1$

Substitute back:

$e^x = 8$ | $e^x = -1$

$x = \ln 8$ | No solution

③ $\sin^2 x - 3\sin x + 2 = 0$ $x = [0, 2\pi)$

let $\sin x = b$

$b^2 - 3b + 2 = 0$

$(b-2)(b-1) = 0$

$b - 2 = 0$ | $b - 1 = 0$

$b = 2$ | $b = 1$

Substitute back

$\sin x = 2$ No solution | $\sin x = 1$ $x = \pi/2$

④ $e^{2x} - 6e^x - 5 = 0$ let $a = e^x$

$a^2 - 6a - 5 = 0$

cannot factor $\Rightarrow$ use quadratic formula!

$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = \frac{6 \pm \sqrt{36 + 20}}{2} = \frac{6 \pm \sqrt{56}}{2}$

$a = \frac{6 + \sqrt{56}}{2}$

$a = \frac{6 - \sqrt{56}}{2} \rightarrow$ Not a solution because < 0

substitute back!

$a = e^x$

$\frac{6 + \sqrt{56}}{2} = e^x$

$x = \ln\left(\frac{6 + \sqrt{56}}{2}\right)$

⑤ $\ln(x+5) + \ln(x-2) = 0$

$\ln((x+5)(x-2)) = 0 \rightarrow$ log/ln rules

$e^{\wedge}$ $e^{\wedge}$

$(x+5)(x-2) = e^0$

$(x+5)(x-2) = 1$

$x^2 + 7x + 10 = 1$

$x^2 + 7x + 9 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{7 \pm \sqrt{49 - 36}}{2} = \frac{7 \pm \sqrt{13}}{2}$

Both positive!

$x = \frac{7 \pm \sqrt{13}}{2}$

⑥ $\left(e^x - \frac{6}{e^x} = 5\right) \cdot e^x$

$e^{2x} - 6 = 5e^x$

$e^{2x} - 5e^x - 6 = 0$

let $y = e^x$

$y^2 - 5y - 6 = 0$

$(y-6)(y+1) = 0$

$y-6=0 \quad | \quad y+1=0$

$y=6 \quad | \quad y=-1$

substitute back:

$e^x = 6 \quad | \quad e^x = -1$

$\ln 6 = x \quad |$ No solution

⑦ $4\sin^3 x - 3\sin x = 0 \qquad x = [0, 2\pi)$

$\sin x \,|\, (4\sin^2 x - 3) = 0$

$\sin x = 0 \quad | \quad 4\sin^2 x - 3 = 0$

$x = 0, \pi \quad | \quad \sin^2 x = 3/4$

$\sin x = \pm\sqrt{3/4} = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

⑧ $\log_4(2x+3) - \log_4(3x+1) = 1$

$\log_4\left(\frac{2x-3}{3x+1}\right) = 1$

$4^1 = \frac{2x-3}{3x+1}$

$\frac{4}{1} = \frac{2x-3}{3x+1}$

$2x-3 = 4(3x+1)$

$2x-3 = 12x+4$

$-7 = 10x$

$x = -7/10$

Not a solution since $3(-7/10)+1 < 0$

No solution

⑨ if $f(x) = 4e^{2x-1}$ find $f^{-1}(x)$

$y = 4e^{2x-1}$

$x = 4e^{2y-1}$

$\frac{x}{4} = e^{2y-1}$

$\ln\left(\frac{x}{4}\right) = 2y-1$

$2y-1 = \ln\left(\frac{x}{4}\right)$

$2y = \ln\left(\frac{x}{4}\right) + 1$

$y = \frac{\ln\left(\frac{x}{4}\right)+1}{2} = f^{-1}(x)$

⑩ If $f(x) = 8^{1+3x}$ find $f^{-1}(x)$

$y = 8^{1+3x}$

$x = 8^{1+3y}$

$\log_8 x = 1+3y$

$y = \frac{\log_8 x - 1}{3}$

$f^{-1}(x) = \frac{\log_8 x - 1}{3}$

⑧ alternative way to solve this one on next page

⑪ if $f(x) = 5\log_4(2x-3)$

$y = 5\log_4(2x-3)$

$x = 5\log_4(2y-3)$

$\frac{x}{5} = \log_4(2y-3)$

$4^{\frac{x}{5}} = 2y-3$

$\frac{4^{\frac{x}{5}}+3}{2} = y$

$f^{-1}(x) = \frac{4^{\frac{x}{5}}+3}{2}$

Note: $x = 8^{1+3y}$

$$\ln x = (1+3y)\ln 8$$

$$\frac{\ln x}{\ln 8} = 1 + 3y$$

$$\frac{\ln x}{\ln 8} - 1 = 3y$$

$$y = \frac{\frac{\ln x}{\ln 8} - 1}{3}$$

Another way to solve #10

(12) let $f(x) = 5\sqrt[3]{4x-1}$
find $f^{-1}(x)$

$$y = 5\sqrt[3]{4x-1}$$

$$x = 5\sqrt[3]{4y-1}$$

$$\frac{x}{5} = \sqrt[3]{4y-1}$$

$$\left(\frac{x}{5}\right)^3 = 4y-1$$

$$\frac{\left(\frac{x}{5}\right)^3 + 1}{4} = y$$

$$f^{-1}(x) = \frac{\left(\frac{x}{5}\right)^3 + 1}{4}$$

(13) If $f(x) = 5x^2 - 3x$ find

$$f(x+h) - f(x)$$

$$f(x+h) = 5(x+h)^2 - 3(x+h)$$

$$f(x+h) - f(x) = 5(x+h)^2 - 3(x+h) - (5x^2 - 3x)$$

$$= 5(x^2 + 2xh + h^2) - 3x - 3h - 5x^2 + 3x$$

$$= \cancel{5x^2} + 10xh + 5h^2 - \cancel{3x} - 3h - \cancel{5x^2} + \cancel{3x}$$

$$= 10xh + 5h^2 - 3h$$